The Auto- and Cross-Dependence of Tail Returns, and Implications for Systemic Risk*

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Abstract

Common systemic risk measures focus on the instantaneous occurrence of triggering and systemic events. However, in this article we show that daily tail stock returns exhibit substantial persistence, auto-serially and cross-serially. In particular, lower tail returns can trigger downward spirals of tail returns. The vulnerability towards downward spirals increases with the institution’s size. To quantify the systemic risk related to tail return spillovers, we develop the Conditional Shortfall Probability (CoSP). We show that CoSP exhibits a substantially smaller estimation error than other systemic risk measures. Our empirical results show that a substantially larger fraction of financial companies exhibits significant tail return spillovers to the American non-financial sector than to the global financial sector. Still, the aggregate spillover risk is larger with respect to the financial market than with respect to non-financial markets. Moreover, the aggregate vulnerability of the overall financial sector towards spillover risk is larger than of single financial subsectors.

Keywords: Investor Sentiment, Tail Dependence, Contagion Period, Return Spillover, Systemic Risk, Financial Markets

JEL Classification: G01, G14, G15, G20

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1 Introduction

Systemic risk is commonly understood as "the risk of collapse of an entire system or entire market, exacerbated by links and interdependencies, where the failure of a single entity or cluster of entities can cause a cascading failure" (Committee on Capital Markets Regulation (2009, p. ES-3)). Thus, systemic risk is connoted with the risk of spillover of adverse events from one or more institutions to a market or system. Systemic risk may be exacerbated, for example, by the spillover of volatility (see Hamao et al. (1990), Diebold and Yilmaz (2009), Diebold and Yilmaz (2012)) or information (see Acharya and Yorulmazer (2002), Wongswan (2006), Ahnert and Bertsch (2015)).

In contrast to these specific channels of contagion, cross-sectional systemic risk measures focus on the spillover of tail returns, as for example in Hartmann et al. (2005), Acharya et al. (2010), Acharya et al. (2012), Adrian and Brunnermeier (2014), Brownlees and Engle (2016). These measures quantify the comovement of tail returns in order to identify institutions that exhibit a particularly large exposure or contribution to a market’s tail risk. Although some studies of systemic risk explicitly account for time-lags between triggering and systemic events, all proposed cross-sectional systemic risk measures assume that tail return spillovers occur instantaneously. However, it seems questionable that a risk measure can determine the directionality of spillovers when tail events materialize instantaneously. This raises the pivotal question, whether spillover effects of tail returns indeed necessarily occur instantaneously.

The purpose of our article is to tackle this question. To comprehensively study tail return dependencies, we assess these on three levels: Firstly, tail auto-dependence of daily stock returns indicates that extreme returns of one institution are followed by tail returns of the same institution. Such an institution is vulnerable towards loss spirals. Secondly, tail cross-dependence of daily stock returns of two institutions shows that loss spirals can also be triggered by a tail return of...
a different institution. Due to the nature of this dependence, we argue that tail cross-serial and auto-dependence is likely to be due to the information processing of investors and their investment behavior.

Finally, we examine the cross-dependence between tail returns of single institutions and different markets, in order to assess the systemic risk associated with these institutions and markets. Thus, our article creates a broader basis for the understanding and measurement of tail return spillovers and systemic risk by comprehensively studying the nature of auto- and cross-dependence of tail returns.

To measure the dependence of tail returns we employ the Conditional Shortfall Probability (CoSP), which is related to the measure of tail-dependence (for example, see McNeil et al. (2015)), and the measure of multivariate extreme bank spillovers in Hartmann et al. (2005). The underlying rationale of CoSP is also similar to ∆CoVaR proposed by Adrian and Brunnermeier (2014). However, our measure exhibits a substantially smaller estimation error than ∆CoVaR. The empirical analysis shows, that approximately 19% of the financial institutions in our sample are significantly systemically important with a non-zero time-lag for the financial sector, while 33% of the financial institutions are significantly systemically important with a non-zero time-lag for the American non-financial sector. Moreover, the overall financial sector is more vulnerable as a whole towards systemic risk than its single subsectors.

Eventually, we contribute to the understanding and measurement of systemic risks in two ways: Firstly, we find that triggering and systemic events may be time-lagged. Secondly, we show how to account for this time-lag in systemic risk measures and, thereby, achieve a notion of econometric causality similar to Granger-causality (see Granger (1969)): If the occurrence of an institution’s tail return at time $t$ and the occurrence of a market’s tail return at time $t + \tau$ are positively dependent, the market’s tail return cannot have had an impact on the institution’s tail return, but only vice versa, as long as stocks are frequently traded. In other words, in this case only a contribution of the institution to systemic risk but not an exposure to systemic risk is plausible.
The correlation between time-lagged returns is an extensively studied phenomenon. In particular, short-horizon portfolio returns are found to be significantly autocorrelated and highly cross-serially correlated (among others, see Fama (1965), Gibbons and Ferson (1985), Conrad and Kaul (1988), Lo and MacKinlay (1990), Boudoukh et al. (1994)). The economic meaning and reason for these correlations have been causing a large debate. Possible explanations include time-varying risk premiums (see Keim and Stambaugh (1984), Conrad and Kaul (1989), Conrad et al. (1991a), Conrad et al. (1991b), Füss et al. (2016)) and market overreactions (see Lehmann (1990), Lo and MacKinlay (1990), Jegadeesh and Titman (1995)).

Furthermore, various studies argue that markets process information rationally, and auto- and cross-correlations are due to market frictions such as measurement errors. Such measurement errors may materialize, for example, due to non-synchronous trading (see Fisher (1966), Cohen et al. (1983), Conrad and Kaul (1988), Lo and MacKinlay (1990), Boudoukh et al. (1994)). Non-synchronous trading periods naturally arise since reported stock prices only reflect the last trade on one specific day. This reported price may deviate from the “true” price and is followed by a non-trading period, which induces a correlation with the next day’s price. Also, market frictions like information, decision, or transaction costs account for a substantial part of the dependence between returns (see Atchinson et al. (1987), Hou and Moskowitz (2005)).

Finally, the high complexity and opaqueness of interconnected markets and institutions, particularly in the financial services sector (see Arora et al. (2009), Moghadam and Viñals (2010), Adrian et al. (2014), Battiston et al. (2015)), but also complex products like hybrid debt, reinsurance, specific forms of parent-subsidiary relationships, captives and other forms of capital transfer mechanisms make it difficult to assess the entire impact of events and information. Time-lags with regard to the information processing of investors are studied by AitSahlia and Yoon (2016), who provide evidence for time-lagged price adjustments prior and after events that contain information about a specific stock. Similarly, Boguth et al. (2016) document significant correlation between average returns at different horizons, and show that slow reaction to market information is an important cause for this finding. Slow diffusion of industry information is particularly present between big and small firms, and driven by sluggish adjustment to negative information (see Hou (2007)).
In conclusion, observed market prices do not necessarily adjust instantaneously to information. Consequently, spillover of tail returns may also occur with a time-lag. This reasoning has already been applied in some studies of contagion and asset pricing. For example, Diebold and Yilmaz (2009) explicitly study volatility spillovers at different time-lags. Furthermore, Branger et al. (2014) derive asset prices if agents are uncertain about the propagation of economic crises. This model is able to reproduce the empirically observed predictability of stock returns. More generally, Aït-Sahalia et al. (2015) model the (time-lagged) contagion of different markets with mutually exciting jump processes.

However, to the best of our knowledge, there is no study about the auto-dependence and cross-serial dependence of tail returns. With our study we aim to fill this gap and, moreover, illustrate implications of our findings for systemic risk measurement. The remainder of the article is organized as follows. In Sections 2 and 3 we study the auto- and cross-dependence of return series, respectively. Thereupon, Section 4 reviews existing systemic risk measures and introduces a new systemic risk measure that incorporates time-lags. In our empirical analysis in Section 5 we apply and compare the new and traditional systemic risk measures. Finally, Section 6 concludes and provides an outlook to future research directions.

2 Auto-Dependence of Returns

In this article, we aim to measure the dependence between two specific events, namely the events that a return is in a specific section $I \subseteq \mathbb{R}$ of the return distribution at time $t$ and at time $t + \tau$. For this purpose, we employ the conditional probability of the respective events, namely

$$\psi_{t,\tau}(I) = \mathbb{P}(r_{t+\tau} \in I_{t+\tau} \mid r_t \in I_t).$$

(1)

In an informationally efficient financial market with rationally behaving market participants and without market frictions and time-varying risk premia, we would expect the time-lagged returns $r_t$ and $r_{t+\tau}$ to be independent (as argued, for example, in Fama (1969) and Billio et al. (2012)). In
the case of independence, \( \psi_{t,\tau}(I) \) equals the reference level \( \mathbb{P}(r_{t+\tau} \in I_{t+\tau}) \). However, the findings of auto- and cross-correlation cited in Section 1 indicate, that, in fact, the return series may be auto-correlated. Thus, we expect that \( \psi_{t,\tau}(I) \neq \mathbb{P}(r_{t+\tau} \in I_{t+\tau}) \) at least for some intervals \( I_t \) and \( I_{t+\tau} \).

A major advantage of this approach in comparison to other studies of extreme dependence (for example see Baur et al. (2012) or Zhu et al. (2015)) is, that it is independent from modeling and distributional assumptions. To cover the full return distribution, we will focus on the conditional probability that a return and the lagged-return are in intervals with the same probability \( q \in (0,1) \):

\[
I^i_t(q) = [\text{VaR}_t(iq), \text{VaR}_t((i + 1)q)],
\]

where \( i = 0, ..., 1/q - 1 \) and \( \text{VaR}_t \) is the Value-at-Risk of returns at time \( t \).\(^3\) To restrict estimation errors but still examine a granular partition of the return distribution, we choose \( q = 5\% \). We study the (time-)unconditional distribution of returns and denote

\[
\psi^i_{\tau}(q) = \mathbb{P}(r_{\tau} \in [\text{VaR}(iq), \text{VaR}((i + 1)q)] | r_0 \in [\text{VaR}(iq), \text{VaR}((i + 1)q)]).
\]

Then, the conditional probability \( \psi^i_{\tau}(q) \) equals the reference level \( q \) if, and only if, the two return events are independent. For illustration purposes, we, firstly, study the auto-dependence of four exemplary institutions, namely AIG, JP Morgan, Qatar International Islamic Bank (in the following: Qatar International), and Comdirect Bank. According to the median market capitalization in the years 1995 to 2015, these represent, respectively, the 99.8\%, 99\%, 55\%, and 50\% largest institution in our sample. In Figure 1 we show the resulting auto-dependence for the four institutions. Each straight line corresponds to the conditional probability for one specific time-lag (in days) with respect to the mid quantile level of the respective interval of the return distribution. If the conditional probability is larger than the reference level \( q \), there is positive auto-dependence.

\(^3\)Note that \( \text{VaR}_t(q) \) corresponds to the return’s quantile at level \( q, F_t^{-1}(q) \).
Clearly, the largest part of the return distribution does not exhibit substantial auto-dependence. However, upper and lower tail returns are not fully independent, but, instead, substantially positively auto-dependent, i.e. likely to persist. Positive auto-dependence of lower tail returns has also been found by Baur et al. (2012). The authors show that lower return quantiles positively dependent on previous returns by means of quantile regression for a time-lag of one day. However, they find that upper quantiles negatively depend on previous returns, but do not distinguish between different levels of previous returns. For example, upper quantiles may negatively depend on previous lower tail returns, but positively depend on previous upper tail returns, as our findings suggest. Then, the overall dependence of upper quantiles on previous returns might still be negative.
Figure 2: Auto-Dependence for median, lower, and upper tail returns of AIG and JP Morgan.
The magnitude of tail auto-dependence is different with respect to the size of the time-lag: As Figure 2 shows, the tail auto-dependence declines for negative or positive tail returns with increasing time-lag. Thus, current and future tail returns become independent for large time-lags.

The reasons for positive auto-dependencies may be the same as for auto-correlations of returns: Measurement errors, non-synchronous trading, market frictions like transaction, information or decision costs, time-varying risk premiums, or psychological factors and investment behavior. However, one can eliminate estimation errors as possible cause, since all considered intervals have the same probability. Thus, (statistical) estimation errors have the same magnitude for each interval. Non-synchronous trading is, also, likely to be independent of the return level and, furthermore, does not affect returns at large time-lags.

Moreover, although market frictions and time-varying risk premiums may depend on return levels in absolute size, they induce a dependence structure independent of return levels.\(^4\)

Therefore, it seems plausible, that unusually large auto-dependencies are caused by investment behavior. Indeed, the auto-dependence of lower or upper tail returns may indicate crisis sentiment or extreme optimism. In general, crisis sentiment is different from investors’ overreaction (as studied, for example, by Lehmann (1990), Jegadeesh and Titman (1995), and Lo and MacKinlay (1990)) in one particular way: If investors overreact at a certain point in time, e.g. heavily sell one security, returns typically experience reversals subsequently, e.g. investors buy the security when prices are low. This type of behavior induces negative auto-correlation. In contrast, crisis sentiment is characterized by a positive auto-dependence, i.e. extremely high losses are likely to be followed by extremely high losses. Such loss spirals may be caused either by a decreasing demand, i.e. decreasing willingness to pay, or increasing supply, i.e. increasing sales. Since both buyer and seller usually have the same information about an asset, loss spirals are likely to be driven by both decreasing demand and increasing supply.

\(^4\)For example, an AR(1)-process leads to a correlation of \(\rho^\tau\) for the returns \(r_t\) and \(r_{t+\tau}\), which is independent of the level of \(r_t\).
Supply driven loss spirals, in particular, are extensively studied in the literature. In this case, the selling of one security lowers its price, which incentivizes other investors to also sell the security, which lowers its price even more. This phenomenon is commonly referred to as fire sales (for example, see Benoit et al. (2016), Allen and Gale (2004), Shleifer and Vishny (1992), Coval and Stafford (2007)). An incentive for fire sales may be given, for example, by risk-based solvency capital requirements: If the value but not the risk of an asset decreases, investors may improve their solvency situation by substituting this asset with a less risky asset.

In order to measure the crisis sentiment of market participants with respect to one particular institution, we focus on the lower tail of its return distribution. The corresponding auto-dependence is measured by the Conditional Shortfall Probability (CoSP), which is the likelihood of a tail return at time $\tau$ given a tail return at time 0, i.e.

$$
\psi_{\tau}(q) = \psi_{\tau}^0(q) = P(r_{\tau} \leq VaR(q) \mid r_0 \leq VaR(q)).
$$

This measure is very similar to the coefficient of lower tail dependency (see Appendix A). The estimation procedure for $\psi_{\tau}(q)$ is outlined in Appendix B.1. A major advantage of CoSP is the availability of a closed-form significance bound, which is given as

$$
k^{*}_{\tau} = \frac{1}{n-\tau q^2} \left( F_{Bin(n-\tau,q^2)}^{-1}(1-\alpha) + 1 \right),
$$

where $\alpha$ is the significance level and $F_{Bin(n-\tau,q^2)}^{-1}$ is the (lower) inverse cumulative distribution of the Binomial distribution (see Appendix B.2). In Figure 3 we show CoSP, a smoothed version of CoSP (see Appendix B.1), the lower confidence bound for a significance level $\alpha = 1\%$, and the reference level $q$ for AIG and JP Morgan. Clearly, both institutions exhibit a significantly positive auto-dependence in the lower tail.

To assess the total crisis sentiment of market participants with respect to one particular institution, we propose to aggregate adverse reactions of market participants over time, conditional on
Figure 3: Lower Tail Auto-Dependence measured by CoSP for AIG and JP Morgan.

\[ \bar{\psi} = \int_1^\infty \psi_\tau(q) - q \, d\tau. \]  

The estimation procedure for \( \bar{\psi} \) is outlined in Appendix B.3. Since \( q \) is the reference level of CoSP, \( \bar{\psi} \) is the Aggregate Excess CoSP.

Our data sample includes historical daily returns of 917 publicly traded financial institutions that are classified as banks (i.e. depository institutions; BAN), brokers (i.e. security and commodity brokers; BRO), or insurers (INS) in Datastream. Moreover, we examine the returns of 36 non-financial companies (NoFIN) that are selected according to a particularly large market capitalization. In Appendix D.2 we report descriptive statistics for the returns of all institutions included in the data sample. All returns are daily for the period from November 21, 1995 to November 20, 2015. We exclude institutions with less than 1750 observations. Then, 725 institutions remain in the sample.

Figure 4 reveals two decisive properties of lower tail auto-dependence with regard to different institutions: Firstly, the Aggregate Excess CoSP is not significantly different for financial subsectors but larger for non-financial companies. Secondly, the Aggregate Excess CoSP increases with the

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\(^5\)The names of the 10 largest institutions in each subsector included in the sample are reported in Table 4.
\(^6\)The non-financial companies’ names are reported in Table 3.
\(^7\)After excluding institutions with too few observations 442 banks, 106 broker, 141 insurer, and 36 non-financial companies remain in the sample.
median market capitalization of the institutions. Thus, non-financial and large institutions are particularly vulnerable to loss spirals.

In addition to the magnitude of auto-dependence, we are also interested in quantifying the average time-lag between lower tail returns. For this purpose, we weight each time-lag with its contribution to the Aggregate Excess CoSP.\(^8\) The corresponding measure is the CoSP-weighted time-lag, which is given as

$$\bar{\tau} = \frac{1}{\bar{\psi}} \int_{1}^{\infty} \tau \left( \psi_{\tau} - q \right) d\tau.$$

The estimation procedure for the CoSP-weighted time-lag is outlined in Appendix B.4. Figure 5 shows, that also the CoSP-weighted time-lag is larger for non-financial companies and increases with the market capitalization. Thus, the average time horizon of loss spirals is larger for large and non-financial companies. Moreover, insurance companies exhibit a slightly larger CoSP-weighted time-lag than other financial institutions.

To sum up, we find significant positive tail auto-dependence for return series, particularly for non-financial and large institutions. The returns of these institutions are particularly vulnerable to

\(^8\)Note the analogies to the duration of Macaulay (1938).
loss spirals, where lower tail returns significantly increase the likelihood of future lower tail returns. Clearly, this type of loss spirals might constitute systemic crises. However, loss spirals might not only be triggered by losses from the considered institutions, but also by return spillovers from other institutions. To study these spillovers, the next section proceeds by studying the cross-serial dependence of returns.

3 Cross-Serial Dependence of Returns

In Figure 6 we show the conditional probability $\psi^i(I)$ for returns of a specific institution conditional on the return of a different institution. Not surprisingly, we also find substantial cross-serial dependence of tail returns. However, the level of lower and upper tail cross-dependence can be very different, and is subject to the direction of spillovers (as, for example, in the case of Qatar International and AIG). Similar to the levels of auto-dependence, the level of cross-dependence also declines for larger time-lags, as Figure 7 shows. Interestingly, the lower tail cross-dependence is not for all institutions significant. In particular, we do not find evidence of significant tail return spillovers from Qatar International to AIG, while AIG exhibits a very small but significant impact on Qatar International. In contrast, there are significant spillovers between AIG and JP Morgan in both directions.
In our data sample, we find that 18% of all possible (directed) links between different institutions exhibit a significant lower tail cross-sectional dependence. In Table 1 we provide the number of links with significant cross-dependence for institutions of different subsectors. The number of significant links is substantially larger for the influence on and from particularly large non-financials (as in our sample). (Depository) banks exhibit the smallest cross-sectional dependence. Moreover, the number of links with significant cross-dependence among insurance companies is the largest among all subsectors.

The declining structure of the lower tail cross-sectional dependence allows to employ the Aggregate Excess CoSP to assess the time-aggregate impact of cross-dependence. Thereby, we focus on differences between the cross-dependence of different subsectors and institutions with different

Figure 6: Cross-Dependence for AIG, JP Morgan, and Qatar International.
levels of market capitalization. Thereby, we are able to test for a lead-lag effect in lower tail returns. The lead-lag effect was originally found by Lo and MacKinlay (1990) for the cross-correlation of returns and implies that large institutions impact small institutions more strongly, than vice versa.

In contrast, Figure 8 reveals that, cross-sectionally, large institutions are more exposed to tail returns than small institutions, regardless of the market capitalization of the triggering institution. Similarly, non-financial companies seem to be more exposed to tail returns of financial institutions than other financial institutions. However, note that our sample of non-financial companies solely consists of 36 particularly large companies. In Section 5 we obtain a smaller exposure of non-financials by studying non-financial markets instead of single and particularly large companies.

Figure 7: Lower Tail Cross-Dependence for AIG, JP Morgan, and Qatar International.
Table 1: Fraction of links between institutions with significant lower tail cross-dependence for different subsectors.

<table>
<thead>
<tr>
<th>From \ To</th>
<th>BAN</th>
<th>BRO</th>
<th>INS</th>
<th>NoFIN</th>
</tr>
</thead>
<tbody>
<tr>
<td>BAN</td>
<td>14.11%</td>
<td>16.08%</td>
<td>18.70%</td>
<td>24.63%</td>
</tr>
<tr>
<td>BRO</td>
<td>16.05%</td>
<td>19.64%</td>
<td>21.93%</td>
<td>32.08%</td>
</tr>
<tr>
<td>INS</td>
<td>18.31%</td>
<td>22.34%</td>
<td>26.07%</td>
<td>37.38%</td>
</tr>
<tr>
<td>NoFIN</td>
<td>25.89%</td>
<td>34.71%</td>
<td>38.99%</td>
<td>77.62%</td>
</tr>
</tbody>
</table>

Figure 8: Lower Tail Cross-Dependence measured by Aggregate Excess CoSP for different Subsectors and Market Capitalizations (in USD). For each box, the central mark is the median, the edges are the 25th and 75th percentiles, $q_1$ and $q_3$, and the maximum whiskers’ length is $1.5(q_3 - q_1)$.
The CoSP-weighted time-lag for tail cross-dependence in Figure 9 indicates, that the total impact of tail returns is more delayed for large insurance and non-financial companies. Similarly to above, with regard to non-financials this result differs from our the analysis in Section 5, when studying non-financial markets instead of single large non-financial companies. However, also in Section 5 we find that insurance companies are exposed to a particularly large time-lag.

The causes for a direct impact of the time $t$-return of institution A for the time $t + \tau$-return of institution B may be similar to the causes for auto-dependence. Furthermore, a positive cross-dependence might also result from the contemporaneous dependence between the returns of different institutions and their auto-dependence. In this case, a tail return from institution A is spilled over to institution B immediately, but affects future returns of institution B due to their positive auto-dependence. For some institutions, the spillover would even trigger a loss spiral as described in Section 2. A similar argument is outlined in Boudoukh et al. (1994) for the cross-correlation of returns.

Eventually, the previous analysis shows that the total impact of tail return spillovers cannot be assessed by taking only the first time period (here: the first day) into account. In contrast, a tail return of institution A often exhibits an adverse effect on the future returns of institution B. In this case, either the tail return of institution A is spilled over immediately and triggers a loss spiral for the returns of institution B, or the tail return of institution A directly triggers a future tail return of institution B.

This finding has important implications for cross-sectional market-based measures of systemic risk, since these measures typically quantify the impact of a lower tail return of one institution on a market’s return. In the next section we review traditional measures for systemic risk. Thereby, we show that CoSP exhibits similarities with several of these traditional systemic risk measures. This finding will motivate the use of the Aggregate Excess CoSP to quantify the aggregate systemic risk of return spillovers in our subsequent empirical analysis.
Figure 9: Time-Lag of Lower Tail Cross-Dependence measured by the CoSP-weighted time-lag for different Subsectors and Market Capitalizations (in USD). For each box, the central mark is the median, the edges are the 25th and 75th percentiles, $q_1$ and $q_3$, and the maximum whiskers’ length is $1.5(q_3 - q_1)$. 
4 Measures for Systemic Risk

4.1 Traditional Global Measures for Systemic Risk

Systemic risk refers to the spillover of adverse events between single institutions and markets. Global cross-sectional systemic risk measures evaluate the spillover of adverse events between one specific institution and market index. Among the global cross-sectional return-based systemic risk measures, there are two particularly popular risk measures: marginal expected shortfall (MES; see Acharya et al. (2010)) and ∆CoVaR (see Adrian and Brunnermeier (2014)). While the MES is the expected return of an institution conditional on market distress, the ∆CoVaR is the change in the market’s tail risk conditional on an institution’s distress. Thus, triggering and systemic event are inverted in both measures. However, Benoit et al. (2013) show, that the systemic risk ranking based on MES is strictly equivalent to a ranking based on the institutions’ betas. Thus, MES reflects systematic (tail-)risk. In the following we aim to shed light on the systemic risk induced by spillovers from single institutions to the market. Thus, we focus on ∆CoVaR.

By conditioning on the triggering event, on first sight it seems that ∆CoVaR is based on a causal relationship between institution and market. However, in common frameworks, ∆CoVaR is the result of the co-movement of (tail-)returns: For example, this can directly be verified in case of bivariate normally distributed returns, for which Adrian and Brunnermeier (2014) show that

\[
\Delta \text{CoVaR} = \sigma^M(-\Phi^{-1}(q))\rho^{I,M},
\]

where $\sigma^M$ is the standard deviation of market returns, $\Phi^{-1}$ is the inverse of the cumulative density function of the standard normal distribution, and $\rho^{I,M}$ is the correlation coefficient between market and institution returns.

More generally, Benoit et al. (2013) find that ∆CoVaR is proportional to the institution’s firm-specific risk if the dependence between financial asset returns is linear. In this case, the pro-

\footnote{Other cross-sectional systemic risk measures are co-risk (see Chan-Lau et al. (2009)) and the distress insurance premium (see Huang et al. (2009)). Bisias et al. (2015) and Benoit et al. (2016) give an overview of these and other systemic risk measures.}
The proportionality coefficient depends on the market’s volatility and the correlation between market’s and institution’s returns. In other words, \( \Delta \text{CoVaR} \) is not able to identify a causal relationship between systemic and triggering event, since it solely focuses on simultaneous events with exceptionally high losses of both the market and institution. Moreover, several studies indicate that the estimation error of \( \Delta \text{CoVaR} \) makes it a rather unreliable systemic risk measure (for example see Castro and Ferrari (2012), Guntay and Kupiec (2014), or Danielsson et al. (2015)).

### 4.2 The Conditional Shortfall Probability

In Section 2 we already introduced the Conditional Shortfall Probability (CoSP) as a measure for the dependence between lower tail returns with a time-lag. In this section we generalize this concept for global cross-sectional systemic risk measurement. Consistent with the dependency-consistent \( \Delta \text{CoVaR} \) proposed by Ergün and Girardi (2013) and Mainik and Schaanning (2014), we interpret the occurrence of one of the \( q^I \cdot 100\% \) smallest institution returns, \( r^I \), as a proxy for a triggering event, and one of the \( q^M \cdot 100\% \) smallest market returns, \( r^M \), as a proxy for a systemic market event.\(^{10}\) To assess the impact of an institution’s triggering event on the market, the Conditional Shortfall Probability (CoSP) is defined by

\[
\psi_\tau(q^M, q^I) = \mathbb{P}(S E^M_\tau \mid T E^I) = \mathbb{P}(r^M_\tau \leq \text{VaR}^M(q^M) \mid r^I \leq \text{VaR}^I(q^I)).
\]  

(9)

Thus, CoSP measures the systemic risk related to a spillover of lower tail returns from an institution to a market. The identification of the VaR-levels \( q^M \) and \( q^I \) is both necessary and challenging: For one exemplary market solely the 1% smallest market returns may relate to a systemic event, while the 5% smallest market returns may be systemic for a different market, for example due to a larger market capitalization. For other markets, that exhibit no systemic risk, \( q^M \) would equal zero. Thus, \( q^M \) and \( q^I \) depend on the respective market’s and institution’s properties.

Clearly, the choice of \( q^M \) and \( q^I \) also depends on the respective definition of systemic risk and triggering events. However, presently there is no common agreement on the definition and level of market-specific systemic risk and institution-specific distress probabilities. For this reason, in

\(^{10}\)Note that the definition of CoSP also allows for other proxies of triggering and systemic events.
the empirical analysis we set (analogously to $\Delta \text{CoVaR}^\leq$) $q^M = q^I = q$, and denote $\psi_{\tau}(q) = \psi_{\tau}(q, q)$.

Several aspects of CoSP can also be found in previous studies of systemic risk. For example, CoSP is inverted to the tail-$\beta$ introduced by Hartmann et al. (2005), which quantifies the vulnerability of single institutions to an extreme negative systematic shock. The idea behind CoSP is also very similar to the idea of the distress spillover measure by Chan-Lau et al. (2012), which assesses extreme changes in banks’ distance to default (DD). Finally, our definition of a triggering event is based on CoVaR$^\leq$. In fact, one might also define a time-lagged CoVaR$^\tau_E$ by

$$
\mathbb{P}(r^M_{\tau} \leq \text{CoVaR}^\tau_E(q) \mid E) = q.
$$

Then, CoVaR$^\tau_E$ and $\psi_{\tau}$ are properties of the same conditional distribution, namely the $q$-quantile and the tail probability. When considering the same market, CoSP and CoVaR$^\tau_E$ also generate the same order of institutions according to systemic risk if the conditional market returns stochastically dominate each other (see Appendix A.2). However, the interpretation for the two measures is different: CoSP captures the likelihood of a predefined stress event (i.e. the size of the conditional distribution’s tail), while CoVaR reflects the tail risk of a triggering event (i.e. the quantile).

Moreover, $\Delta \text{CoVaR}$ depends on market volatility: If market returns are more volatile, the change in tail risk, as measured by $\Delta \text{CoVaR}$, is larger.\footnote{For the Gaussian case the impact of spillover effects measured by $\Delta \text{CoVaR}$ are solely driven by market volatility, as Equation (8) shows.} Consequently, the systemic risk implied by $\Delta \text{CoVaR}$ is larger on more volatile markets, ceteris paribus. Nonetheless, systemic events like market distress or failure have important implications beyond lost market returns. For example, a small return loss on a largely capitalized market may be more adverse than a large return loss on a less capitalized market. In addition, the distress of one market may also impact other markets and industries, but also affect political and socioeconomic dimensions. For example, Chan-Lau (2010) suggests to regulate too-connected-to-fail institutions based on societal losses. Boyd and Heitz (2016) study the cost to the macro-economy due to increased systemic risk triggered by too-big-to-fail banks.
Therefore, we suggest to predetermine a maximum level of systemic market returns that corresponds to a systemic event, i.e. adjust $q^M$ to the level of a market’s systemic risk probability. Then, CoSP is able to quantify the risk of such a systemic event due to tail return spillovers, and, thereby, is independent from market volatility.

CoSP has several advantages from a statistical point of view: For example, asymptotic confidence bounds for CoSP are available in closed form, which permits to assess the statistical significance of CoSP in a straightforward manner. Moreover, the standard error of CoSP is smaller then the standard error of $\Delta \text{CoVaR}^\leq$ (see Appendix C.1). Thus, less data is needed to estimate CoSP. Consequently, CoSP exhibits a larger reliability than $\Delta \text{CoVaR}^\leq$ in the sense of Danielsson et al. (2015), as we show in Appendix C.2 and in Figure 10. In Appendix A we discuss several other properties of CoSP.

![Graphs](image1.png)

(a) Mean Absolute Percentage Error (MAPE) of $\hat{\psi}_0$ and $\Delta \text{CoVaR}^\leq$ for student-distributed returns.

(b) Reliability of CoSP and $\Delta \text{CoVaR}^\leq$.

Figure 10: Estimation Error and Reliability of CoSP and $\Delta \text{CoVaR}^\leq$ for a sample size of 2500 (a description of the error and reliability measure can be found in Appendix C.2).

4.3 Aggregate Systemic Risk and the CoSP-weighted Time-Lag

Since both the speed of decline and amplitude of CoSP reflect the persistence of an institution’s triggering event, we suggest to assess the total systemic impact of triggering events with the
Aggregate Excess CoSP as introduced in Section 2, i.e.

$$\bar{\psi}_0 = \int_0^\infty \psi_\tau(q^M, q^I) - q^M \, d\tau.$$  \hspace{1cm} (11)

For systemic risk measurement we include simultaneous spillovers with zero time-lag $$\tau = 0$$, to quantify the total impact of return spillovers. By definition the Aggregate Excess CoSP reflects the total extent to which a triggering event of an institution increases the likelihood of a systemic event. A second measure we already introduced in Section 2 is the CoSP-weighted time-lag, which is

$$\bar{\tau}_0 = \frac{1}{\bar{\psi}_0} \int_0^\infty \tau \left( \psi_\tau(q^M, q^I) - q^M \right) \, d\tau.$$  \hspace{1cm} (12)

This measure explicitly focuses on the timing dimension of systemic events. In particular, it is an average of all time-lags, which are weighted with their contribution to the Aggregate Excess CoSP. A major advantage of $$\bar{\tau}$$ is, that it is measured in time units (e.g. days).

### 4.4 The Contagion Period

Another quantity of interest is the contagion period $$\tau^*$$ that lies between a triggering event and the first subsequent (or simultaneous) systemic market reaction. We study a discrete approximation of the probability distribution of the contagion period $$\tau^*$$, which is\(^{12}\)

$$F_{\tau^*}(x) = \mathbb{P}(\tau^* = x) = \mathbb{P}\left( r^M_x \leq VaR^M(q^M) \text{ and } r^M_0, \ldots, r^M_{x-1} > VaR^M(q^M) \mid r^I \leq VaR^I(q^I) \right).$$  \hspace{1cm} (14)

In the empirical analysis we examine the median value $$\tau^*_{0.5}$$, i.e. the median contagion period between triggering and first subsequent (or simultaneous) systemic event.

\(^{12}\)Note that $$F_{\tau^*}$$ is indeed a probability distribution if almost surely at some point in time a systemic market event occurs, which seems reasonable. In this case,

$$1 = \mathbb{P}\left( \bigcup_{x \geq 0} \left\{ r^M_x \leq VaR^M(q^M) \text{ and } r^M_0, \ldots, r^M_{x-1} > VaR^M(q^M) \mid r^I \leq VaR^I(q^I) \right\} \right) = \sum_x F_{\tau^*}(x).$$  \hspace{1cm} (13)
5 Empirical Analysis of Systemic Tail Return Spillovers

In this section we apply the methodology as described in the previous section and examine the aggregate systemic risk and time-lag of systemic risk for our data sample, as introduced in Section 2. As described in Section 4.2 we set $q^M = q^I = q$ for the systemic risk and institution distress probability. We set $q = 1\%$ as reference level to account for particularly adverse triggering and systemic events. To compute the lower bound of significance we use the significance level $\alpha = 1\%$.

To avoid endogeneity, we compute our own market indices by excluding the currently considered institution. The computational procedure is described in Appendix D.1. In Appendix D.2 in Figure 19 (a) we show the resulting indices for banks (BAN), brokers (BRO), insurers (INS) and the whole financial market (FIN) if no institution is excluded from the index. Furthermore, we consider three continent-specific indices for non-financial companies, namely indices for the Americas (AMC), Asia (ASIA), and Europe (EU), which are shown in Appendix D.2 in Figure 19 (b).

![Graphs showing CoSP triggered by exemplary institutions](image)

Figure 11: CoSP triggered by exemplary institutions w.r.t. the FIN index.
In Figure 11 we show the CoSP with respect to the financial index for several institutions that exhibit a typical pattern. More examples are shown in Appendix E and can be provided by the authors on request. As for the lower tail cross-dependence in Section 3, CoSP is declining and converges to the reference level $q$ for $\tau \to \infty$. Furthermore, for Wells Fargo, Blackrock, and Metlife CoSP is significantly larger than the reference level for several lags $\tau > 0$. Thus, there is statistically significant influence between triggering events and systemic events with a time-lag. However, there are substantial differences between the institutions: For the exemplary non-financials in Figure 11 and Appendix E CoSP declines relatively fast and is rather small, whereas for financial institutions CoSP seems to be larger and more slowly declining.

We begin our cross-sectional analysis by examining the dependence between the measures of co-movement, in particular CoSP without time-lag, i.e. $\psi_0$, MES, and $\Delta \text{CoVaR}^<=$.\(^{13}\) Across all observations with respect to the financial index the correlation between $\psi_0$ and MES is $-80.76\%$, and between $\psi_0$ and $\Delta \text{CoVaR}^<=$ it is $-76.5\%$.\(^{14}\) The levels of correlation are very similar with respect to other indices. Thus, all three measures $\psi_0$, MES and $\Delta \text{CoVaR}^<=$ result in very similar levels of systemic risk. Also, all three measures exhibit a large correlation with the institution’s $\beta$, as given by $\rho^{I,M} \frac{\sigma^I}{\sigma^M}$, ranging between 68\% for $\Delta \text{CoVaR}^<=$ and 95.5\% for MES.

However, the correlation is substantially smaller if considering the Aggregated Excess CoSP $\bar{\psi}_0$ instead of $\psi_0$. For the aggregate systemic risk with respect to the financial index the correlation with MES is $-50.39\%$ and the correlation with $\Delta \text{CoVaR}^<=$ is $-54.71\%$. For other indices the values are similar. In particular, the correlation with $\beta$ is only 45.05\%. In Figure 25 in Appendix E we also show scatter plots that assess the dependence between MES, $\Delta \text{CoVaR}^<=$, $\bar{\psi}_0$, and $\beta$ for our full data sample. We conclude that the Aggregate Excess CoSP is able to capture a dimension of systemic risk that traditional systemic risk measures of co-movement like MES or $\Delta \text{CoVaR}^<=$ are not able to fully reflect. In particular, the Aggregate Excess CoSP exhibits a smaller dependence on systematic risk, as measured by $\beta$.

\(^{13}\)For all measures we set the VaR-levels equal to 1\%. The results can be found in Table 6 in Appendix E.

\(^{14}\)Note that, by definition, the larger MES and $\Delta \text{CoVaR}^<=$ are the smaller is the anticipated systemic risk, whereas for $\psi_0$ a large value indicates a large systemic risk. Therefore, a negative correlation of the measures’ values indicates a positive relationship with respect to the level of systemic risk.
Secondly, we exclude the number of institutions that do not exhibit significant lagged systemic return spillover risk. Thereby, we classify an institution as significantly systemically important due to tail return spillovers (s.s.i.) if \( H(\tau) \geq k^*_\tau \) for at least one lag \( \tau > 0 \). According to this criterion, roughly 19% of the financial institutions in the sample are significantly systemically important for the financial sector. Interestingly, more financial institutions are s.s.i. for the American non-financial sector (33%), while less of the financial institutions are s.s.i. for the European (16%) and Asian non-financial sector (19%). Moreover, a substantially larger fraction of insurance institutions is classified as significantly systemically important for the financial sector (29%) than of brokers (21%) and banks (15%). The fraction of significantly systemically important institutions for each (sub-)sector with respect to the different indices is shown in Appendix E in Figure 24.

Thirdly, we focus on the measures for aggregate tail return spillovers, the contagion period, and systemic time-lags. In Figure 12 we show the distribution of the Aggregate Excess CoSP for all s.s.i. institutions with respect to the indices for banks (BAN), brokers (BRO), insurers (INS), and the American non-financial index. The Aggregate Excess CoSP triggered by s.s.i. non-financial companies is the smallest with respect to all indices, in particular with respect to the banking, brokerage, and American non-financial market. Moreover, the median Aggregate Excess CoSP triggered by s.s.i. brokers is the largest, while the Aggregate Excess CoSP triggered by s.s.i. insurance institutions is similar to that of s.s.i. banks with respect to the brokerage and American non-financial market. In contrast, insurer trigger a larger Aggregate Excess CoSP than banks w.r.t. the banking sector but a smaller w.r.t. the insurance sector

This finding is similar to the results of Billio et al. (2012) and Adrian and Brunnermeier (2014). However, it is in contrast with the results of Cummins and Weiss (2014), who argue that systemic risk within the insurance industry is considerably smaller than between insurance and banking industry. Also, in contrast to Chen et al. (2013), we find that the median Aggregate Excess CoSP triggered by s.s.i. insurers with respect to the banking market is slightly larger than vice versa.\(^{15}\)

\(^{15}\)The median Aggregate Excess CoSP triggered by s.s.i. banks w.r.t. the insurance market is 5.07, whereas the median Aggregate Excess CoSP triggered by s.s.i. insurers w.r.t. the banking market is 5.41.
### (a) Aggregate Excess CoSP w.r.t. the BAN index.

### (b) Aggregate Excess CoSP w.r.t. the BRO index.

### (c) Aggregate Excess CoSP w.r.t. the INS index.

### (d) Aggregate Excess CoSP w.r.t. the American NoFIN index.

Figure 12: Aggregate Excess CoSP w.r.t. the BAN, BRO, INS and American NoFIN indices triggered by significantly systemically important institutions of the subsectors BAN, BRO, INS and NoFIN. For each box, the central mark is the median, the edges are the 25th and 75th percentiles, $q_1$ and $q_3$, and the maximum whiskers’ length is $1.5(q_3 - q_1)$.

However, the opposite is true for the upper quantiles of the Aggregate Excess CoSP. Thus, median tail spillovers are larger from s.s.i. insurers to banks than vice versa, while extremely larger tail spillovers are larger from s.s.i. banks to insurers than vice versa.

Interestingly, the ranking of subsectors is substantially different if studying the $\Delta$CoVaR$^\leq$ instead of the Aggregate Excess CoSP, as shown in Figure 26 in Appendix E. $\Delta$CoVaR$^\leq$ indicates that non-financial companies trigger the largest systemic risk, particularly w.r.t. the insurance, brokerage, and American non-financial market. Banks are found to trigger the smallest systemic risk. This finding is analogous to Guntay and Kupiec (2014), who criticize $\Delta$CoVaR and MES.
for indicating that non-financial companies pose a larger systemic risk than financial institutions. Thus, $\Delta \text{CoVaR} \leq$ would imply the regulation of systemically important non-financial institutions rather than of systemically important financial institutions (SIFIs). This finding is not only in opposition to the regulation of SIFIs (see Financial Stability Board (2009)), but also challenges economic intuition.

To study a market’s exposure towards systemic risk (i.e. its susceptibility), Figure 13 depicts the distribution of the Aggregate Excess CoSP by comparing the systemic risk triggered by s.s.i. institutions of one subsector with respect to different indices. In general, we find that the systemic risk with respect to non-financial markets is smaller than for the financial sector. Clearly, the Aggregate Excess CoSP with respect to the Asian non-financial market is the smallest for all subsectors, whereas the Aggregate Excess CoSP with respect to the European non-financial market is slightly larger than for the American non-financial market. Among the financial sector, depository banks have the smallest exposure to systemic risk triggered by other banks. In particular, banks trigger a larger systemic risk with respect to brokers, insurance companies, and the overall financial market. Moreover, we find that the intra-subsector systemic risk of depository banks is smaller than the intra-subsector systemic risk of brokers. Interestingly, Figure 13 also indicates that the overall financial sector as a whole is more exposed to the systemic risk caused by tail return spillovers than single subsectors. Thus, the interconnectedness of single subsectors contributes to a larger vulnerability towards systemic risk.

Next, we examine the contagion period and CoSP-weighted time-lag. To this end, we start with studying the median contagion period between all stress (triggering) events of significantly systemically important institutions (s.s.i.) and the first (subsequently or simultaneously) occurring systemic market event. Figure 14 clearly shows that the median contagion period is different from zero for more than 75% of all s.s.i. institutions with respect to all indices. Thus, not all institutions’ stress events immediately trigger a systemic market event. In general, the banking, insurance, and overall financial markets show systemic distress after a median time of approximately 5 days, subsequently to a triggering event of s.s.i. financial institutions. In contrast, the median contagion period for triggering events of s.s.i. financial institutions is approximately 10 days for systemic
Figure 13: Aggregate Excess CoSP w.r.t. different financial indices BAN, BRO, INS, FIN and non-financial indices AMC, ASIA, EU, triggered by significantly systemically important banks, brokers, insurers, and non-financial companies. For each box, the central mark is the median, the edges are the 25th and 75th percentiles, $q_1$ and $q_3$, and the maximum whiskers’ length is $1.5(q_3 - q_1)$.

The median contagion period measures the time-lag between stress events of an institution and a market. Still, the respective stress events might occur randomly. Therefore, we also study the CoSP-weighted time-lag, i.e. the weighted average time-lag between triggering and systemic events,
whereby the weighting factors are the contribution of the single time-lags to the Aggregated Excess CoSP. In Figure 15 we show the distributions of the CoSP-weighted time-lag for systemically important institutions. In contrast to the median contagion period, the CoSP-weighted time-lag is smaller with respect to non-financial markets than for financial markets. Thus, it takes longer to observe a systemic event on non-financial markets after a triggering event, whereas the systemic risk associated with these large time-lags is rather small. If triggered by banks, insurers, or non-financial institutions, the CoSP-weighted time-lag is particularly large with respect to the insurance market.

Most interestingly, the CoSP-weighted time-lag indicates a systemic risk cascade: If systemic risk is triggered by banks, insurers, or non-financial companies, firstly banks are systemically affected, then brokers, and, lastly, insurance companies. In contrast, if systemic risk is triggered by brokers, firstly brokers are systemically affected, then insurers, and, lastly, banks. A rationale for this cascade may be the investment behavior of the three types of financial institutions: Since banks are particularly exposed to the distress of other institutions due to loan activities, they are the first to be infected by the distress of borrowers. Since brokers tend to invest on a shorter time horizon than insurance companies, the distress of other institutions might infect brokers earlier than insurance companies.
(a) CoSP-weighted time-lag triggered by banks.

(b) CoSP-weighted time-lag triggered by brokers.

(c) CoSP-weighted time-lag triggered by insurers.

(d) CoSP-weighted time-lag triggered by non-financials.

Figure 15: CoSP-weighted time-lag w.r.t. different financial indices BAN, BRO, INS, FIN and non-financial indices AMC, ASIA, EU, triggered by significantly systemically important banks, brokers, insurers, and non-financial companies. For each box, the central mark is the median, the edges are the 25th and 75th percentiles, $q_1$ and $q_3$, and the maximum whiskers’ length is $1.5(q_3 - q_1)$.

6 Conclusion

Since cash-flows and information may take time to spread within and across (financial) markets, systemic risk is not only the risk of simultaneously occurring extreme events. It is also the risk of systemic market (participants’) reactions that occur with a time-lag to the triggering event. With this in mind, our article is twofold: Firstly, we study the level of auto- and cross-serial dependence of returns, particularly in the lower tail. Secondly, we apply our findings for the measurement of systemic risk. Therefore, our article creates a broader basis for the understanding and measurement of tail return spillovers and systemic risk.
The dependence structure of returns reveals the persistence of tail returns: A tail return at time $t$ increases the likelihood of a tail return at time $t + \tau$. The corresponding positive auto-dependence of lower tail returns indicates the vulnerability of certain institutions to loss spirals of returns. This vulnerability increases with the market capitalization of the institution. The positive cross-dependence of lower-tail returns shows that the spillover of a tail return of institution A at time $t$ increases the likelihood of a tail return of institution B at time $t + \tau$.

Motivated by these results, we argue that cross-sectional return-based systemic risk measures should incorporate time-lagged tail return spillovers. For this purpose, we propose to employ the Conditional Shortfall Probability (CoSP), which is the likelihood that a systemic market event occurs $\tau$ days after the triggering event of an institution. CoSP exhibits a smaller estimation error and a larger reliability than $\Delta$CoVaR. In typical cases CoSP is exponentially declining and converges to the reference level. Based on CoSP, we define two aggregate risk measures: The Aggregate Excess CoSP, $\bar{\psi}_0$, and the CoSP-weighted time-lag, $\bar{\tau}_0$. The Aggregate Excess CoSP reflects the overall systemic influence of an institution on the market. The CoSP-weighted time-lag is the weighted average of all time-lags. Together, these aggregate measures capture the overall persistence of systemic spillover risk of tail returns triggered by an institution.

In the empirical analysis we study all institutions with significant systemic spillover risk in the global subsectors banks, brokers, insurance, and non-financial companies. Hence, we are among the first to differentiate systemic risk between and within the financial sector and the real industry. In particular, we find that 33% of the financial institutions in our sample are significantly systemically important for the American non-financial sector, whereas only 19% of the institutions are significantly systemically important for the (global) financial sector.

However, the results show that the systemic risk triggered by systemically important institutions, as measured by the Aggregate Excess CoSP, is larger with respect to financial markets than for non-financial markets. In particular, the Aggregate Excess CoSP is very small with respect to the Asian non-financial market, and particularly large with respect to the overall financial market.
Interestingly, the Aggregate Excess CoSP triggered by banks is larger with respect to the insurance and brokerage market than with respect to the banking market. Moreover, insurance institutions exhibit a comparable level of the Aggregate Excess CoSP to that of banks.\textsuperscript{16}

By studying the systemic contagion period we find that approximately 10 days after a stress event of systemically important financial institutions we can observe a systemic event on the brokerage or non-financial markets. This time-lag is only approximately 5 days for the banking, insurance, and overall financial market. In contrast, the systemic risk associated with large time-lags with respect to the non-financial market is rather small, as the CoSP-weighted time-lag shows. Moreover, the CoSP-weighted time-lag reveals a systemic cascade effect for the financial sectors: Systemic risk triggered by banks, insurers, or non-financial companies tends to influence brokers firstly, secondly banks, and, thirdly, insurance institutions. The different investment behavior of the different types of institutions may serve as a reason for this observation.

Our findings create a basis for further research in various forms. For example, disentangling the effects of auto- and cross-serial dependence might provide more detailed insights into the causes of tail return spillovers. Moreover, the drivers for systemic risk and contagion periods may be fundamentals that need to be identified in order to understand the underlying rationale of the timing dimension of systemic risk.

\textsuperscript{16}This result is similar to Billio et al. (2012) and Adrian and Brunnermeier (2014), but is contrary to Cummins and Weiss (2014) and Berdin and Sottocornola (2015).
Appendix

A Properties of CoSP

The conditional shortfall probability (CoSP) is given as
\[ \psi_{\tau}(q^M, q^I) = P(r^M_\tau \leq VaR^M(q^M) \mid r^I \leq VaR^I(q^I)) . \] (15)

Thus, \( \psi_{\tau}(q) = \psi_{\tau}(q, q) \) is very similar to the coefficient of lower tail dependence. In particular, the latter is the limit of \( \psi_{\tau}(q) \) as \( q \) approaches 0, i.e.
\[ \lambda_{\tau} = \lim_{q \to 0^+} \psi_{\tau}(q), \] (16)
where \( \lambda_{\tau} \) is the coefficient of lower tail dependence between \( r^I \) and \( r^M_\tau \) (see McNeil et al. (2015, p.247)).

A.1 Symmetry

In general, we have
\[ \psi_{\tau}(q^M, q^I) = P(SE^M_\tau \mid TE^I) = P(r^M_\tau \leq VaR^M(q^M) \mid r^I \leq VaR^I(q^I)) \] (17)
\[ = \frac{P(SE^M_\tau, TE^I)}{q^I} = \frac{q^M}{q^I} P(TE^I \mid SE^M_\tau) . \] (18)

Thus, \( \psi_{\tau}(q^M, q^I) \) and \( P(TE^I \mid SE^M_\tau) \) are proportional, whereas \( \psi_{\tau}(q, q) \) is equal to \( P(TE^I \mid SE^M_\tau) \).

If \( \tau > 0 \), the latter probability, \( P(TE^I \mid SE^M_\tau) \), cannot be interpreted in a causal sense, i.e. \( SE^M_\tau \) can not have caused \( TE^I \) since it happened later in time. Still, \( P(TE^I \mid SE^M_\tau) \) is the likelihood that the institution exhibits an extraordinarily small return \( \tau \) days before a systemic market event \( SE^M_\tau \). From this perspective, \( \psi_{\tau}(q) \) may also be interpreted as the likelihood of a triggering event of a specific institution given a systemic market event.

In contrast, the symmetry of \( \psi_0(q) \) is very reasonable, since it is the result of co-movements between \( r^M \) and \( r^I \). In other words, one can in general not identify a causal relationship between the
events. This co-movement is also reflected in other systemic risk measures like MES or $\Delta \text{CoVaR}^{-}$ in the sense that these are proportional to the institution’s firm-specific risk if the dependence between financial asset returns is linear.\footnote{This is a main finding of Benoit et al. (2013).}

A.2 Systemic Risk Rankings of CoSP and $\Delta \text{CoVaR}$

$\text{CoVaR}_{E}(q)$ is defined as the Value-at-Risk (VaR) of the conditional distribution of the market return $r^{M}$, i.e.

$$\mathbb{P}(r^{M} \leq \text{CoVaR}_{E}(q) \mid E) = q.$$  \hspace{1cm} (19)

Then, $\Delta \text{CoVaR}$ is the difference between the market’s CoVaR conditional on a triggering event $TE^{I}$ and a benchmark event $BM^{I}$, i.e.

$$\Delta \text{CoVaR} = \text{CoVaR}_{TE^{I}}(q) - \text{CoVaR}_{BM^{I}}(q).$$  \hspace{1cm} (20)

Adrian and Brunnermeier (2014) define the triggering event as the institution’s return being at the VaR($q$), i.e. $TE^{I} = \{r^{I} = \text{VaR}^{I}(q)\}$, and the benchmark event as the institution’s return being at the median state, i.e. $BM^{I} = \{r^{I} = \text{VaR}^{I}(0.5)\}$, which yields

$$\Delta \text{CoVaR}^{-}(q) = \text{CoVaR}_{r^{I}=\text{VaR}^{I}(q)}(q) - \text{CoVaR}_{r^{I}=\text{VaR}^{I}(0.5)}(q).$$  \hspace{1cm} (21)

To also incorporate more severe losses than VaR($q$), Ergün and Girardi (2013) propose

$$\Delta \text{CoVaR}^{\leq}(q) = \text{CoVaR}_{r^{I}\leq\text{VaR}^{I}(q)}(q) - \text{CoVaR}_{r^{I}\in[\mu^{I} \pm \sigma^{I}]}(q),$$  \hspace{1cm} (22)

where $\mu^{I}$ and $\sigma^{I}$ are the mean and standard deviation for the return of the institution, respectively. The change in the triggering event definition from being exactly at the VaR to being at or below the VaR also effects the consistency of CoVaR: Mainik and Schaanning (2014) show that $\text{CoVaR}_{r^{I}\leq\text{VaR}^{I}(q)}(q)$ is a continuous and increasing function of the dependence parameter between $r^{I}$ and $r^{M}$, while $\text{CoVaR}_{r^{I}=\text{VaR}^{I}(q)}(q)$ is not.
In the following we examine the relationship between two different returns $r_{I1}$ and $r_{I2}$ and a market return $r_M$. For simplicity, we focus on the case with $q^M = q^I = q$. Under the assumption that $r_M^\tau | r_{I1} \leq VaR_{I1}^I(q)$ first-order stochastically dominates $r_M^\tau | r_{I2} \leq VaR_{I2}^I(q)$, i.e. for all $x \in \mathbb{R}$

$$
\mathbb{P}(r_M^\tau \leq x | r_{I1} \leq VaR_{I1}^I(q)) \leq \mathbb{P}(r_M^\tau \leq x | r_{I2} \leq VaR_{I2}^I(q)),
$$  

(23)

we have $\psi_{I1}^\tau(q) \leq \psi_{I2}^\tau(q)$. Moreover, for $\text{CoVaR}^\tau$ we have

$$
\text{CoVaR}_{r_{I1} \leq VaR_{I1}^I(q)}^\tau(q) \geq \text{CoVaR}_{r_{I2} \leq VaR_{I2}^I(q)}^\tau(q).
$$  

(24)

Hence, with respect to both risk measures $I_2$ is more systemically important than $I_1$. Also, if the market risk conditional on the benchmark events is approximately equal, i.e. $\text{CoVaR}_{BM_{I1}}^\tau(q) \approx \text{CoVaR}_{BM_{I2}}^\tau(q)$, for $\Delta \text{CoVaR}_{I1}^\tau(q)$ we have

$$
\Delta \text{CoVaR}_{I1}^{\tau} \geq \Delta \text{CoVaR}_{I2}^{\tau}. 
$$  

(25)

The condition that $r_M^\tau | r_{I1} \leq VaR_{I1}^I(q)$ first-order stochastically dominates $r_M^\tau | r_{I2} \leq VaR_{I2}^I(q)$ can often be observed for financial return series. In Figure 16 we show two exemplary empirical cumulative density functions (ecdf) for lag $\tau = 0$ for the unconditional and conditional returns of the financial index. Particularly in the lower tail $r_M^\tau | r_{I1} \leq VaR_{I1}^I(0.01)$ stochastically dominates $r_M^\tau | r_{I2} \leq VaR_{I2}^I(0.01)$.

\section*{B Estimation Procedure}

\subsection*{B.1 Estimation of CoSP}

An estimator for $\psi_{I1}^\tau$ is given as

$$
\hat{\psi}_{I1}^\tau = \frac{1}{q(n-\tau)} \sum_{t=1}^{n-\tau} 1 \{ r_t^I \leq \hat{VaR}_t^I(q^I), r_t^M + \tau \leq \hat{VaR}_t^M(q^M) \},
$$  

(26)

\footnote{Note that stochastic dominance in the lower tail is sufficient to obtain the same order of the institutions.}
where the Value-at-Risk estimate is the \([nq^x]-\)th smallest observation for return \(r^x\), \(\hat{VaR}^x(q^x) = r^{(\lfloor q^xn \rfloor)}\). To estimate \(\psi_\tau(q^M, q^I)\) we employ historical simulation (HS).\(^{19}\) The use of this simplified approach is particularly motivated by the fact that, as to our knowledge, this is the first study about the interdependence of lagged tail-returns. Thus, it seems unreasonable to impose distributional or modeling assumptions.\(^{20}\) Additionally, it seems intuitive that systemic market events mostly occur in times with large volatility. In other words, the maximum return level that corresponds to systemic market distress, \(VaR^M(q^M)\), should not depend on the current volatility level but on the (time-)unconditional volatility. Therefore, we employ the (time-)unconditional Value-at-Risk to estimate CoSP.

To smooth the estimation error of \(\hat{\psi}_\tau(q)\), we suggest to fit \(\hat{\psi}_\tau(q)\) for \(\tau = 1, 2, \ldots\) to the following function:

\[
H(\tau) = q^M + e^{-a\tau^2 + b\tau + c}, \quad \text{with } a > 0, b, c \in \mathbb{R}.
\]  

In other words, we assume that \(\psi_\tau\) declines exponentially and find this confirmed in the empirical

\(^{19}\)There exist several studies discussing and improving the statistical properties of HS and other estimation approaches for time series of returns, for example Danielsson and Zhou (2015), Hendricks (1996), Hull and White (1998), Kuester et al. (2006) or Pritsker (2001). However, these focus on quantile or moment estimation and do not consider a time-lag and, thus, cannot be applied for the estimation of CoSP.

\(^{20}\)Note, that by employing HS we do not need to assume that the full bivariate return distribution is stationary. In contrast, it is sufficient to assume that solely the dependence between lagged tail returns is stationary over time.
analysis. Note that we do not include co-movements at the time-lag $\tau = 0$ in the fitting procedure, since these do not reflect persistence and, particularly, do not steadily continue $\psi_\tau$, as the results in Section 5 suggest. The choice of $H$ has several advantages: Since $H$ is always larger than $q^M$ but converges to $q^M$, we only capture systemically important lags, i.e. fit the part of $\hat{\psi}_\tau(q^M, q^I)$ that lies above $q^M$. However, if $\hat{\psi}_\tau(q^M, q^I) < q^M$ for many lags $\tau$, we have $H \approx q^M$, indicating that the systemic risk is zero.

B.2 Lower Bound of Significance for $\hat{\psi}_\tau$

Denote by $n_\tau = n - \tau$ the number of available observations for lag $\tau$. Under the assumption that $\mathbb{1}\{r_{t+\tau}^M \leq \text{VaR}^M(q^M), r_t^I \leq \text{VaR}^I(q^I)\}$ are iid for $t = 1, \ldots, n_\tau$, it follows

$$\sum_{t=1}^{n_\tau} \mathbb{1}\{r_{t+\tau}^M \leq \text{VaR}^M(q^M), r_t^I \leq \text{VaR}^I(q^I)\} \sim \text{Bin}(n_\tau, \hat{\psi}_\tau q^I),$$

(28)

where $\text{Bin}(n, p)$ is the Binomial distribution. Hence, under the null hypothesis $H_0 : \psi_\tau = q^M$, i.e. that systemic event $SE_\tau^M$ and triggering event $TE^I$ are independent, we have

$$n_\tau q^I \hat{\psi}_\tau \sim \text{Bin}(n_\tau, q^M q^I).$$

(29)

The null hypothesis if $\hat{\psi}_\tau \geq k^*_\tau$ is rejected with a significance level of $\alpha \in (0, 1)$. Thus, a lower bound for the rejection area, $k^*_\tau$, can be computed as follows:

$$\alpha = \mathbb{P}_{H_0} \left( \hat{\psi}_\tau \geq k^*_\tau \right) = \mathbb{P}_{H_0} \left( \sum_{t=1}^{n_\tau} \mathbb{1}\{r_{t+\tau}^M \leq \text{VaR}^M(q^M), r_t^I \leq \text{VaR}^I(q^I)\} \geq n_\tau q^I k^*_\tau \right)$$

(30)

$$= 1 - F_{\text{Bin}(n_\tau, q^M q^I)}(n_\tau q^I k^*_\tau - 1)$$

(31)

$$\Leftrightarrow \quad 1 - \alpha = F_{\text{Bin}(n_\tau, q^M q^I)}(n_\tau q^I k^*_\tau - 1)$$

(32)

$$\Leftrightarrow \quad n_\tau q^I k^*_\tau - 1 = F_{\text{Bin}(n_\tau, q^M q^I)}^{-1}(1 - \alpha)$$

(33)

$$\Leftrightarrow \quad k^*_\tau = \frac{1}{n_\tau q^I} \left( F_{\text{Bin}(n_\tau, q^M q^I)}^{-1}(1 - \alpha) + 1 \right),$$

(34)

where $F_{\text{Bin}(n_\tau, q^M q^I)}^{-1}$ is the (lower) inverse cumulative distribution of the Binomial distribution.
By accounting for estimation errors, we employ the fitted CoSP, $H(\tau)$, instead of $\hat{\psi}_\tau$ to assess the significance of systemic importance. Therefore, an institution is classified as significantly systemically important if $H(\tau) \geq k^*_\tau$ for at least one time-lag $\tau > 0$. We do not consider co-movements at $\tau = 0$, since these are solely due to comovement. In contrast, for non-zero time-lags triggering events cannot be caused by systemic events.

B.3 Estimation of the Aggregate Excess CoSP

To account for estimation errors, we employ the fitted CoSP $H$ (as described in Section B.1) for lags $\tau \geq 1$ to estimate the Aggregate Excess CoSP. For the CoSP at lag $\tau = 0$ we include $\hat{\psi}_0$. Then, the estimator for the Aggregate Excess CoSP is given as

$$\bar{\psi}_0 = \hat{\psi}_0(q^M, q^I) - q^M + \int_1^\infty H(\tau) - q^M \, d\tau. \tag{35}$$

Firstly, note that

$$\int H(\tau) - q^M \, d\tau = \int e^{-a\tau^2 + b\tau + c} \, d\tau = e^{c + \frac{b^2}{4a}} \sqrt{\frac{\pi}{4a}} \text{erf} \left( \frac{2a\tau - b}{2\sqrt{a}} \right). \tag{36}$$

Therefore, if $a > 0$,

$$\int_1^T e^{-a\tau^2 + b\tau + c} \, d\tau = e^{c + \frac{b^2}{4a}} \sqrt{\frac{\pi}{4a}} \left( \text{erf} \left( \frac{2aT - b}{2\sqrt{a}} \right) - \text{erf} \left( \frac{2a - b}{2\sqrt{a}} \right) \right) \tag{37}$$

and

$$\bar{\psi}_0 = \left( \hat{\psi}_0(q^M, q^I) - q^M \right) + e^{c + \frac{b^2}{4a}} \sqrt{\frac{\pi}{4a}} \left( 1 - \text{erf} \left( \frac{2a - b}{2\sqrt{a}} \right) \right), \tag{38}$$

since $\lim_{x \to \infty} \text{erf}(x) = 1$. However, if $a = 0$, we have

$$\int H(\tau) - q^M \, d\tau = \int e^{b\tau + c} \, d\tau = \frac{1}{b} e^{b\tau + c}, \tag{39}$$
thus, if \( b < 0 \),

\[
\int_1^T e^{b\tau + c} d\tau = \frac{1}{b} \left( e^{bT+c} - e^{b+c} \right)
\]  

(40)

and

\[
\bar{\psi}_0 = \left( \hat{\psi}_0(q^M, q^I) - q^M \right) - \frac{e^{b+c}}{b}.
\]  

(41)

### B.4 Estimation of the CoSP-weighted time-lag

To account for estimation errors, we employ the fitted CoSP \( H \) (as described in Section B.1) for lags \( \tau \geq 1 \) to estimate the CoSP-weighted time-lag. Then, the estimator for the CoSP-weighted time-lag is given as

\[
\bar{\tau} = \frac{1}{\bar{\psi}} \int_1^\infty \tau (H(\tau) - q^M) d\tau.
\]  

(42)

Firstly, note that

\[
\int \tau (H(\tau) - q^M) d\tau = \int \tau e^{-a\tau^2 + b\tau + c} d\tau = \frac{1}{4a^{3/2}} \left( \sqrt{\pi} b e^{\frac{b^2}{4a}} \text{erf}\left( \frac{2a\tau - b}{2\sqrt{a}} \right) - 2\sqrt{ae}^{\tau(b-a\tau)+c} \right).
\]  

(43)

Therefore,

\[
\bar{\tau} = \frac{1}{\bar{\psi}} \lim_{L \to \infty} \frac{1}{4a^{3/2}} \left( \sqrt{\pi} b e^{\frac{b^2}{4a}} \text{erf}\left( \frac{2aL - b}{2\sqrt{a}} \right) - 2\sqrt{ae}^{L(b-aL)+c} \right)
\]  

(44)

\[- \frac{1}{\bar{\psi}4a^{3/2}} \left( \sqrt{\pi} b e^{\frac{b^2}{4a}} \text{erf}\left( \frac{2a - b}{2\sqrt{a}} \right) - 2\sqrt{ae}^{b-a+c} \right)
\]  

(45)

\[= \frac{1}{4a^{3/2}\bar{\psi}} \left( \sqrt{\pi} b e^{\frac{b^2}{4a}} \left( 1 - \text{erf}\left( \frac{2a - b}{2\sqrt{a}} \right) \right) + 2\sqrt{ae}^{b-a+c} \right).
\]  

(46)

However, if \( a = 0 \), we have

\[
\int \tau (H(\tau) - q^M) d\tau = \int \tau e^{b\tau + c} d\tau = \left( \frac{\tau}{b} - \frac{1}{b^2} \right) e^{b\tau + c},
\]  

(47)
thus, if \( b < 0 \),

\[
\tau = \frac{1}{\psi} \int_{1}^{\infty} \tau e^{br+c} d\tau \\
= \frac{1}{\psi} \left[ \lim_{L \to \infty} \left( \frac{L}{b} - \frac{1}{b^2} \right) e^{bL+c} - \left( \frac{1}{b} - \frac{1}{b^2} \right) e^{b+c} \right] = \frac{1 - b}{\psi b^2} e^{b+c}.
\]  

(48)

(49)

C Standard Errors and Reliability of \( \Delta \text{CoVaR} \) and CoSP

In this section we examine the standard errors and reliability of HIS estimators for \( \Delta \text{CoVaR} \leq \) and \( \psi_\tau \). For simplicity, we focus on lag \( \tau = 0 \) (i.e. co-movements), since the computation and results for all other lags are equivalent. As in the previous sections, the VaR-level is set to 1% for both measures.

C.1 Standard Errors

Firstly, we perform a Monte-Carlo analysis in two steps: In the first step, we study the mean absolute percentage errors (MAPE) of the risk measures for returns that are student t-distributed. To this end, we estimate the covariance matrix of the firm’s and financial index’ returns from our data sample by means of the method of moments (see Section D.2).\(^{21}\) Then, we draw samples from the student-distribution by employing the Cholesky composition of the resulting covariance matrix. The number of samples per iteration of the Monte-Carlo algorithm is set to the maximum number of observations in our data set, which is \( n = 5219 \), but we also study the implications of a smaller sample size of \( n = 2500 \). For \( N \) realizations (Monte-Carlo iterations) for the estimator \( \hat{\vartheta} \) the mean absolute percentage error (MAPE) of the estimator is given as (for example see Tsay (2010, p.217))

\[
\text{MAPE} = \frac{1}{N} \sum_{i=1}^{N} \left| \frac{\hat{\vartheta}_{i}^{(n)} - \bar{\vartheta}(n)}{\bar{\vartheta}(n)} \right|,
\]

(50)

where \( \hat{\vartheta}_{i}^{(n)} \) is the \( i \)-th realization of the estimator (either \( \Delta \text{CoVaR} \leq \) or \( \hat{\psi}_0 \)) and \( \bar{\vartheta}(n) \) the average realized value of the estimator. The MAPE can be interpreted as the average absolute deviation relative to the true value of \( \vartheta \). Since the latter is not known, we approximate this true value by \( \bar{\vartheta}(n) \).

\(^{21}\)In line with Section D.2 we set \( \sqrt{\text{var}(\tau)} = 0.0236 \), \( \sqrt{\text{var}(\tau_{M})} = 0.013 \) and the correlation to 0.25.
Figure 17: MAPE of $\widehat{\Delta \text{CoVaR}}^\leq$ and $\hat{\psi}_0$ for student-distributed returns.

We show the resulting MAPE for different degrees of freedom (which correspond to the tail-size of the distribution) in Figure 17. Clearly, the MAPE of $\hat{\psi}$ is substantially smaller than the MAPE of $\widehat{\Delta \text{CoVaR}}^\leq$. Interestingly, for very small degrees of freedom (i.e. a very heavy tail) the estimation error for $\widehat{\Delta \text{CoVaR}}^\leq$ is particularly large and decreases with increasing degrees of freedom, while the estimation error for $\hat{\psi}_0$ is particularly small for small degrees of freedom.

As a second step, we apply a nonparametric bootstrap algorithm to draw samples from the historical returns of exemplary institutions and the financial index. As before, the sample size in each bootstrap step is set to $n = 5219$ and we take $N = 100000$ bootstrap samples. In Table 2 we show the resulting MAPE for $\widehat{\Delta \text{CoVaR}}^\leq$, $\text{CoVaR}_{r,t \leq \text{VaR}(0.01)}$, $\text{CoVaR}_{r,t \in [\mu_t \pm \sigma_t]}$ and $\hat{\psi}_0$. Clearly, the estimation error of $\hat{\psi}_0$ is substantially smaller for all considered institutions. Moreover, $\text{CoVaR}_{r,t \leq \text{VaR}(0.01)}$ has an enormously large estimation error: For some institutions the mean absolute error is 100 times as large as the mean value of the systemic risk measure. This result highly questions the use of $\text{CoVaR}_{r,t \leq \text{VaR}(0.01)}$ and, thus, is in line with the findings of Castro and Ferrari (2012), Danielsson et al. (2015) and Guntay and Kupiec (2014).
\[
\Delta\text{CoVaR} \leq \text{CoVaR}_{r,I} \leq \text{VaR}(0.01) \ni \left[ \mu_I \pm \sigma_I \right] \ni \psi_0
\]

<table>
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<tr>
<th></th>
<th>$\Delta\text{CoVaR} \leq$</th>
<th>$\text{CoVaR}_{r,I} \leq \text{VaR}(0.01)$</th>
<th>$\text{CoVaR}_{r,I} \in [\mu_I \pm \sigma_I]$</th>
<th>$\psi_0$</th>
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Table 2: MAPE of $\Delta\text{CoVaR} \leq$, $\text{CoVaR}_{r,I} \leq \text{VaR}(0.01)$, $\text{CoVaR}_{r,I} \in [\mu_I \pm \sigma_I]$ and CoSP for bootstrap samples of size $n = 5219$.

### C.2 Reliability

In this section we compare the reliability of $\Delta\text{CoVaR} \leq$ and $\psi_0$. For this purpose we employ the framework proposed by Danielsson et al. (2015). In this framework an institution is classified as causing systemic risk if its probability to be among the most risky institutions is larger than 90% according to a given systemic risk measure. Then, the reliability of this risk measure is given as the fraction of institutions identified as guilty among all most risky institutions. More specifically, in the baseline calibration 10% of the institutions are assumed to be most risky. Thus, the reliability is given as the fraction of institutions identified to be among the most 10% risky institutions with a probability larger than 90%. For a motivation for this framework and details regarding the computational implementation we refer to Danielsson et al. (2015).

To compare the reliability of $\Delta\text{CoVaR} \leq$ and CoSP we mainly follow the calibration of Danielsson et al. (2015): The reliability is computed for returns in rolling windows of 5 years, for each window we only consider the 200 firms with the highest market capitalization at the end of the window, and 10% of the institutions are assumed to be guilty. However, for our bootstrap algo-
algorithm we use a simple non-parametric bootstrap with 100000 iterations and blocks with sample sizes of 2500 or 5219 observations. The resulting reliability of the risk measures is shown in Figure 18. Clearly, CoSP is substantially more reliable than $\Delta\text{CoVaR}^\leq$, particularly in the case of less data.

![Figure 18: Reliability of CoSP and $\Delta\text{CoVaR}^\leq$.](image)

**Figure 18:** Reliability of CoSP and $\Delta\text{CoVaR}^\leq$.

### D Data and Methodology

#### D.1 Market Indices

To account for endogeneity of publicly available market indices, i.e. the issue that institutions may already be incorporated in the index, we compute own market indices similarly to Chan-Lau (2010). To this end, we denote by $MC_t^{(i)}$ the market capitalization of institution $i$ at time $t$, i.e. $MC_t^{(i)} = P_t^{(i)} \cdot \text{Shares}_t^{(i)}$, where $P_t^{(i)}$ is the stock price and $\text{Shares}_t^{(i)}$ the number of shares at time $t$. Moreover, by $TR_t^{(i)}$ we denote the total (dividend-adjusted) return index of institution $i$.\(^{22}\) A market is denoted by a subset $S \subseteq \{1, ..., M\}$, i.e. the institutions that are included in the market. Then, the index for market $S$ excluding institution $j$ is given as the weighted average of the total return indices:

$$INDEX_t^{S \setminus j} = INDEX_{t-1}^{S \setminus j} \sum_{s \in S \setminus \{j\}} \frac{MC_{t-1}^{(s)}}{\sum_{s \in S \setminus \{j\}} MC_{t-1}^{(s)}} \frac{TR_t^{(s)}}{TR_{t-1}^{(s)}}. \quad (51)$$

\(^{22}\)The total return index reflects the evolution of the stock price assuming that dividends are re-invested to purchase additional units of equity.
To adjust for different currencies, we calculate the market capitalization in US dollar. Therefore, the time $t$ price of institution $s$ is given by

$$P_t^{(s)} = \tilde{P}_t^{(s)}/ER_t^{(s)},$$

(52)

where $\tilde{P}_t^{(s)}$ is the time $t$ price in currency $\tilde{C}$ and $ER_t^{(s)}$ is the exchange rate from currency $\tilde{C}$ to US Dollar at time $t$. Finally, the market return is computed as

$$r^M_t = r_t^{S|j} = \log \left( \frac{INDEX_t^{S|j}}{INDEX_{t-1}^{S|j}} \right).$$

(53)

**D.2 Data and Descriptive Statistics**

![Figure 19: Financial and non-financial indices.](image)

(a) Financial indices.  
(b) Non-financial indices.
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<th>$SRR^{BRO}$</th>
<th>$SRR^{INS}$</th>
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Table 3: Non-financial companies included in the data sample sorted according to market capitalization. $SRR^M$ corresponds to the rank of a company among all non-financial companies according to the Aggregate Excess CoSP w.r.t. the market $M$. If the respective company is not significantly systemically important, the corresponding cell is left empty.
Table 4: Names of the ten largest institutions (by market capitalization in November 2015) in each subsector.

<table>
<thead>
<tr>
<th>Index</th>
<th>No. of institutions</th>
<th>$\tau$</th>
<th>$\sqrt{\text{var}(\tau)}$</th>
<th>$r_{0.1}$</th>
<th>$r_{0.5}$</th>
<th>$r_{0.9}$</th>
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</thead>
<tbody>
<tr>
<td>BAN</td>
<td>567</td>
<td>2.29e-05</td>
<td>0.013</td>
<td>-0.015</td>
<td>0.000</td>
<td>0.014</td>
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<tr>
<td>BRO</td>
<td>151</td>
<td>2.85e-04</td>
<td>0.014</td>
<td>-0.015</td>
<td>0.001</td>
<td>0.015</td>
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<tr>
<td>INS</td>
<td>199</td>
<td>6.36e-05</td>
<td>0.015</td>
<td>-0.015</td>
<td>0.000</td>
<td>0.015</td>
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<tr>
<td>FIN</td>
<td>917</td>
<td>4.17e-05</td>
<td>0.013</td>
<td>-0.014</td>
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<td>0.014</td>
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<tr>
<td>AMC NoFIN</td>
<td>1265</td>
<td>2.56e-04</td>
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<td>-0.013</td>
<td>0.001</td>
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<tr>
<td>ASIA NoFIN</td>
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<td>0.000</td>
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<td>-0.013</td>
<td>0.001</td>
<td>0.013</td>
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Table 5: Mean, standard deviation and quantiles of different index returns.
Figure 20: Distribution of mean and standard deviation of returns, and correlation between institution returns and returns of the financial index.
E Additional Figures

Figure 21: CoSP triggered by exemplary banks and brokers w.r.t. the FIN index.
Figure 22: CoSP triggered by exemplary brokers and insurance institutions w.r.t. the FIN index.
Figure 23: CoSP triggered by exemplary non-financial companies w.r.t. the FIN index.

Figure 24: Fraction of significantly systemically important institutions among the subsectors FIN, BAN, BRO, INS w.r.t. the BAN, BRO, INS, FIN, American, Asian and European non-financial index.
Table 6: Empirical correlation between different (systemic) risk measures at level $q = 1\%$ with respect to the FIN index across the full sample.

<table>
<thead>
<tr>
<th>$\psi_0$</th>
<th>-MES</th>
<th>$-\Delta \text{CoVaR}^\leq$</th>
<th>$\beta$</th>
<th>$\text{VaR}^I$</th>
<th>AggExcCoSP $\bar{\psi}_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>80.76%</td>
<td>76.50%</td>
<td>76.74%</td>
<td>-6.94%</td>
<td>68.02%</td>
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<tr>
<td>-MES</td>
<td>67.61%</td>
<td>95.47%</td>
<td>32.54%</td>
<td>50.39%</td>
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<tr>
<td>$-\Delta \text{CoVaR}^\leq$</td>
<td>68.10%</td>
<td>6.06%</td>
<td>54.71%</td>
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<td></td>
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<tr>
<td>$\beta$</td>
<td></td>
<td></td>
<td>29.80%</td>
<td></td>
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<tr>
<td>$\text{VaR}^I$</td>
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<td>-10.69%</td>
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</tbody>
</table>

Figure 25: Scatter Plots of MES, $\Delta \text{CoVaR}^\leq$, $\bar{\psi}_0$, and $\beta$ for all institutions and indices.
Figure 26: $\Delta \text{CoVaR} \leq$ w.r.t. the BAN, BRO, INS, and American NoFIN indices triggered by significantly systemically important institutions of the subsectors BAN, BRO, INS and NoFIN. For each box, the central mark is the median, the edges are the 25th and 75th percentiles, $q_1$ and $q_3$, and the maximum whiskers’ length is $1.5(q_3 - q_1)$. 
References


