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Privacy Concerns in Insurance Markets: Implications for Market Equilibria and Social Welfare

Irina Gemmo, Mark J. Browne, Helmut Gründl**

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Abstract

Telemonitoring devices can be used to screen consumer characteristics and mitigate information asymmetries that lead to adverse selection in insurance markets. Nevertheless, some consumers value their privacy and dislike sharing private information with insurers. In a secondbest efficient Miyazaki-Wilson-Spence (MWS) framework, we allow consumers to reveal their risk type for an individual subjective cost and show analytically how this affects insurance market equilibria as well as social welfare. We find that information disclosure can substitute deductibles for consumers whose transparency aversion is sufficiently low. This can lead to a Pareto improvement of social welfare. Yet, if all consumers are offered cross-subsidizing contracts, the introduction of a screening contract decreases or even eliminates cross-subsidies. Given the prior existence of a cross-subsidizing MWS equilibrium, utility is shifted from individuals who do not reveal their private information to those who choose to reveal. Our analysis informs the discussion on consumer protection in the context of digitalization. It shows that new technologies challenge cross-subsidization in insurance markets, and it stresses the negative externalities that digitalization has on consumers who are unwilling to take part in this development.

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^{*}Corresponding author: telephone: +49 69 798 33694, e-mail: gemmo@finance.uni-frankfurt.de

^{**}Irina Gemmo and Helmut Gründl are affiliated with the International Center for Insurance Regulation at Goethe University Frankfurt, Faculty of Economics and Business Administration, House of Finance, PF39, Theodor-W.-Adorno-Platz 3, 60629 Frankfurt am Main, Germany. Mark J. Browne is the Robert Clements Distinguished Chair and the Department Chair of the School of Risk Management in the Tobin College of Business at St. John's University, New York. This paper was previously entitled "Transparency Aversion and Insurance Market Equilibria". The authors are grateful for helpful comments from Arnaud Goussebaile, Mike Hoy, Mirko Kraft, Christian Kubitza, Wanda Mimra, Olivia S. Mitchell, Alexander Mürmann, Francois Pannequin, Richard Peter, Casey Rothschild, Hato Schmeiser, Jan-Hendrik Weinert and seminar participants at the University of Pennsylvania, the University of Oxford, ETH Zurich, and St. John's University New York, as well as participants at the Meeting of the German Finance Association 2017, the Seminar of the European Group of Risk & Insurance Economists 2017, the Meeting of the American Risk and Insurance Association 2017, the Risk Theory Society Seminar 2017, and the Congress of the German Insurance Science Association 2017. Irina Gemmo thanks the W. R. Berkley Corporation for supporting her work with a Berkley Fellowship at the School of Risk Management, Insurance, and Actuarial Science at St. John's University. Any errors are our own.

1 Introduction

Collecting information for risk classification can mitigate information asymmetries that lead to adverse selection and moral hazard in insurance markets. Insurers are increasingly using new technologies to acquire information to improve their underwriting. Telemonitoring devices, such as wearables in health insurance and telematics systems in motor insurance, are examples. In September 2018, the life insurer *John Hancock* announced it would leave the traditional life insurance model behind and exclusively sell life insurance policies that come with its *Vitality* program. With this program, John Hancock offers discounts on premiums, other rewards and a free wearable in return for data on the insured's behavior and health status. Individuals can calculate their vitality age on its website by answering questions about eating habits, hours of exercising, smoking habits, alcohol intake, height, weight, waist circumference, blood pressure, cholesterol and mental well-being.¹ The French insurer Axa offers consumers a discount on the insurance premium in exchange for the installation of a Drivebox in the vehicle that records driving behavior on the basis of four criteria: forced acceleration, sudden brakes, high-speed corners, and speed.²

The value of private information has been subject to public discussion about consumer protection, for instance in the context of the recent Facebook-Cambridge Analytica data scandal³ and The General Data Protection Regulation (GDPR) (EU) 2016/679, implemented in 2018.⁴ Many consumers value their privacy and do not feel comfortable sharing too much information with public institutions or firms such as insurers.⁵ Some exhibit a disutility from giving up privacy. The degree of these privacy concerns can differ across consumers but it does not necessarily depend on whether consumers are "low" or "high" risk. It is instead related to the value they place on privacy, which depends, for example, on their views on topics such as digitalization, cyber security, and consumer rights, as well as trust in firms and public institutions with respect to data abuse, and even their

¹See John Hancock (2017). A person's *Vitality Age* is believed to serve as an indicator for overall health and inform the insurer about a person's mortality in a more comprehensive way than does chronological age.

 $^{^{2}}$ See Axa (2015).

³See for instance New York Times (2018).

⁴See European Parliament (2016).

⁵See for instance Actuaries Institute (2016). The debate on privacy has even reached the non-academic fiction literature. In the dystopian novel *The Method* (Zeh (2012)), the German author Juli Zeh describes a future health dictatorship, where laws are written in order to optimize population health.

political orientation.⁶ The disutility a consumer experiences depends on his or her preference for privacy and may outweigh the potential economic benefit achieved by disclosing information to an insurer in order to qualify for a lower premium.

This chapter analyzes how an insurer's introduction of a contract that requires disclosure of private information affects insurance market equilibria and social welfare. In our model, insureds can choose between this contract or one that does not require revealing private information. We show analytically how this affects the standard results regarding the second-best efficient insurance market equilibria within the Miyazaki-Wilson-Spence (MWS) framework. Our analysis shows that revealing private information can result in a greater level of insurance coverage for some insureds. We show that the availability of a screening contract, which requires revealing private information, can lead to a Pareto improvement in social welfare and a Pareto superior market allocation, if the fraction of high risks in the market without a screening contract is sufficiently high for the market equilibrium to be described by non-cross-subsidizing contracts in the Rothschild-Stiglitz sense. If a cross-subsidizing MWS equilibrium exists, the availability of a screening contract decreases or even eliminates cross-subsidies. The resulting equilibrium then depends on the fraction of low risks with privacy concerns in the market. The premium for an insurance policy that does not require policyholders to reveal private information then depends on the availability of an insurance contract that does require this information, as well as on the number of consumers choosing such a contract. Given the prior existence of a cross-subsidizing MWS equilibrium, the availability of a screening contract results in less coverage for low risks with privacy concerns, and high risks pay a higher premium for full coverage. Utility is shifted from individuals who do not reveal their private information to those who choose to reveal. In this case, the impact a screening contract has on the insurance market's performance as well as on social welfare is ambiguous, and it depends on the composition of individuals in the market with respect to their risk type and privacy concerns.

We further look at how the existence of privacy concerns in insurance markets affects consumer utility and social welfare, by comparing these when some consumers value their privacy sufficiently

⁶Muermann et al. (2019) use a telematics data set of driving behavior and the corresponding drivers' insurance data set to analyze the relevance of private information of driving behavior for policyholders' choice of car insurance contracts and the conditional loss distributions. They find that the choice of a telematics based insurance contract is correlated with policyholder characteristics. Such a pay-as-you-go policy is more likely to be chosen by young women living in urban areas. While this result might reflect the own risk assessment of this consumer group, it might also give some indication on this consumer group's privacy concerns.

high to reject the offer of a screening contract, versus when no consumer values privacy. If privacy concerns lead to cross subsidization, utility is shifted from low risks, who exhibit these privacy concerns, to high risks. Yet, low risks' loss of utility stemming from their privacy concerns is higher in the case in which the market equilibrium does not involve cross-subsidization, since the protection of private information then requires them to forgo a substantial amount of insurance coverage. With respect to monetary utility, privacy concerns harm those who exhibit them. Privacy concerns can improve the utility of high risks, and they can be welfare enhancing when there are few high risks in the pool and hardly any low risks are willing to share their information.

In the context of consumer protection, our analysis provides a theoretical foundation for the negative externalities that digitalization has on consumers who are unwilling to take part in this development. It shows that new technologies challenge cross-subsidization in insurance markets, and the policies offered to each consumer depend on other consumers' valuation of private information. In order to mitigate the disadvantages that low risks have from valuing their private information, possible approaches could be to try minimizing privacy concerns, e.g., with better data protection, using a redistribution scheme, or implementing stricter regulation with respect to data usage for pricing.

Section 2 provides a literature review. In Section 3, we introduce the theoretical framework of our model. Section 4 presents the equilibria that emerge when introducing the fairly priced full coverage insurance policy that requires the revelation of private information. Resulting implications on utilitarian social welfare are analyzed in Section 5. Section 6 analyzes the effects that privacy concerns have on consumers' expected utility and social welfare given that such a policy exists. Section 7 discusses some of the assumptions made and provides suggestions for potential future research. Section 8 concludes.

2 Related Literature

We build on the standard adverse selection literature. The widely-referenced study by Rothschild and Stiglitz (1976) (RS) analyzes insurance market equilibria in the context of perfectly competitive insurers and two types of consumers: individuals with a high probability of loss, and individuals with a low loss probability. Insurers cannot observe consumer risk types. The market equilibrium outcomes in this model depend on the fraction of high-risk individuals in the market. If this fraction exceeds a critical value, a pooling contract priced at the average risk does not attract low-risk consumers and therefore the market equilibrium is described by two self-selecting separating contracts without any cross-subsidies between the risk types. If the fraction of high risks is less than the critical value, there is no Nash equilibrium because competitors could always attract low risks with a more attractive contract. Wilson (1977) modifies the assumptions of the RS model to allow an insurer to anticipate which policies offered by competitors will become unprofitable as a result of changes in its own policies. He assumes that unprofitable policies will be withdrawn. The insurer adjusts its supply accordingly or withdraws its own policies if they, in turn, become unprofitable. This property ensures the existence of an equilibrium. If a separating equilibrium in the sense of Rothschild and Stiglitz (1976) exists, the Wilsonian equilibrium equals the RS separating equilibrium. Otherwise, the market is described by a Wilsonian pooling equilibrium. In either case, the market equilibrium is not efficient in terms of risk allocation, since low-risk individuals receive only partial coverage. Miyazaki (1977) and Spence (1978) extend the Wilsonian anticipatory equilibrium analysis to contract menus that result in separating, cross-subsidizing, jointly zero-profit making Miyazaki-Wilson-Spence contracts that are second-best efficient.

Several studies explore how screening policyholders' characteristics can mitigate inefficient information asymmetries (e.g., Crocker and Snow (1986), Crocker and Snow (1992), Crocker and Snow (2011), Dionne and Rothschild (2014), and Bijlsma et al. (2017)). Hendren (2013) focuses on the role of insurance buyers' private information in insurance rejections. Some articles (e.g., Hoy (1982), Hoy (1984), Hoy (2006), and Finkelstein et al. (2009)) analyze the implications of screening and categorization on social welfare. Browne and Kamiya (2012) examine a framework in which consumers can purchase an insurance policy that requires taking an underwriting test and sharing the results with the insurer. In a Wilsonian market with nonmyopic insurers, they show that offering such policies leads to the existence of underwriting equilibria in which low-risk individuals obtain greater insurance coverage than they would in a setting without an underwriting test. The authors consider a positive fee for the underwriting test but do not take into account consumers' valuation of privacy.

Filipova-Neumann and Welzel (2010) name two potential reasons for disliking the revelation of private information: (1) The premium risk that individuals face if they are not informed about their own risk type, and (2) The inherent disutility from giving up privacy. While several studies have analyzed the first case, often in the context of medical checkups or genetic testing (e.g., Doherty and Thistle (1996), Doherty and Posey (1998), Hoy and Polborn (2000)), interest in the second case has been increasing as well. Beresford et al. (2012), Benndorf et al. (2015), Feri et al. (2016), Benndorf and Normann (2018), Schudy and Utikal (2017), and Marreiros et al. (2017), among others, analyze individuals' privacy preferences in various experimental settings. Gradwohl and Smorodinsky (2017) model the impact of players' privacy concerns on their choice of actions in strategic settings in a variant of signaling games. The authors focus on the effects on pooling behavior, misrepresentation of information, and inefficiency. Acquisti et al. (2016, p.483) point out that "exploiting the commercial value of data can often entail a reduction in private utility, and sometimes even in social welfare overall." Among other personal costs, they list quantity discrimination in insurance markets, the risk of identity theft, and "the disutility inherent in just not knowing who knows what or how they will use it in the future." Filipova-Neumann and Welzel (2010) examine the effects of monitoring technologies in automobile insurance markets with adverse selection. In addition to the usual second best contract, they introduce a contract that gives the insurer access to recorded information after an accident. The authors show that offering this kind of monitoring technologies can lead to a Pareto improvement of social welfare in an automobile insurance market with asymmetric information. In one scenario, Filipova-Neumann and Welzel (2010) account for privacy concerns that are represented by a loss of utility for a fraction of low risks that is defined as having an inherent preference for privacy.⁷ In their model, the preference for privacy does not change their main result. Yet, in their setting, data are retrieved and analyzed only when the driver reports an accident.⁸ Therefore, an ex-ante classification of risks is not possible. The adjustment to the respective risk type revealed by the 'black box' is displayed as an ex-post adjustment of the indemnity payment rather than as an ex-ante premium adjustment.

In what follows, we analyze how the existence of insurance contracts that include screening possibilities with respect to consumers' risk types affects the standard results in insurance markets

⁷They analyze privacy concerns in a more extensive way in a previous version of this article (Filipova et al. (2005)), where they also consider individuals that are uninformed about their own risk type.

⁸This setting can impact the correlation between the probability of an accident and privacy concerns. Low-risk drivers with privacy concerns could exhibit lower private costs from having such a black box installed than implied by their level of privacy concerns, since they know that the likelihood of having to report their data is small, if data are not reported at any time but only in the case of an accident.

with nonmyopic insurers. To the best of our knowledge, this is the first analysis to examine the role of privacy preferences in such as setting, as well as their effects on market equilibria and social welfare.

3 Theoretical Framework

3.1 Basic Framework

We consider an imperfect insurance market with asymmetric information. Individuals are endowed with initial wealth w_0 and face a loss of $D \in [0, w_0]$ with probability π_i , where $i \in \{L, H\}$ and $0 < \pi_L < \pi_H < 1$, i.e., individual *i* is either of the low-risk type *L* or of the high-risk type *H*. The loss probability is an individual's private information. The fraction of high-risk individuals in the market is denoted by λ , and $(1-\lambda)$ is the fraction of low risks. Individuals are risk-averse with a twice differentiable concave CRRA utility function over final wealth u(w). Risk neutral, nonmyopic insurers operate in a competitive market environment and offer jointly zero-profit making insurance policies that are characterized by an indemnity payment *q* offered in return for a premium *p* paid by the policyholder.⁹ As a result, an individual's monetary wealth is given by $w_1 = w_0 - p$ if no loss occurs, whereas the realization of a loss yields a wealth state $w_2 = w_0 - p + q - D$.

Consumers' private information can be collected, for instance, through the implementation of technological monitoring devices. Each individual decides whether to reveal private information before contract offers are made. That is, coverage q and premium p are determined by anticipating the resulting effect of the coverage on the premium, given the information shared. Consumers then choose whether to purchase the insurance product according to their individual expected utility.

3.2 Standard Policies

Let α_i with $i \in \{H, L\}$ denote the RS separating contract for high risks and low risks, respectively. In the non-cross-subsidizing RS separating equilibrium (α_H, α_L) with $\alpha_H = (p^{\alpha_H}, D)$ and $\alpha_L = (p^{\alpha_L}, q^{\alpha_L})$, contracts break even individually and the insurance premium is given by $p^{\alpha_i} = \pi_i q^{\alpha_i}$. Low risks forgo utility because they do not receive full insurance coverage.¹⁰ A

⁹For the sake of simplicity, we neglect insurers' acquisition and administrative expenses.

¹⁰With perfect information and actuarial fair pricing, individuals would always prefer full insurance to partial insurance (Mossin's Theorem). In the non-cross-subsidizing RS separating equilibrium, however, it must hold that

high-risk individual's expected utility of a non-cross-subsidizing RS separating contract α_H is given by:

$$V_H(\alpha_H) = (1 - \pi_H) \cdot u(w_0 - p^{\alpha_H}) + \pi_H \cdot u(w_0 - p^{\alpha_H} + D - D)$$

= $u(w_0 - \pi_H D).$ (1)

A low-risk individual's expected utility of a non-cross-subsidizing RS separating contract α_L is given by:

$$V_L(\alpha_L) = (1 - \pi_L) \cdot u(w_0 - p^{\alpha_L}) + \pi_L \cdot u(w_0 - p^{\alpha_L} + q^{\alpha_L} - D)$$

= $(1 - \pi_L) \cdot u(w_0 - \pi_L q^{\alpha_L}) + \pi_L \cdot u(w_0 - \pi_L q^{\alpha_L} + q^{\alpha_L} - D).$ (2)

Miyazaki-Wilson-Spence (MWS) equilibrium contracts (β_H, β_L) are characterized by $\beta_i = (p^{\beta_i}, q^{\beta_i})$ with $i \in \{H, L\}$. An individual's expected utility with MWS contract β_i is given by:

$$V_i(\beta_i) = (1 - \pi_i) \cdot u(w_0 - p^{\beta_i}) + \pi_i \cdot u(w_0 - p^{\beta_i} + q^{\beta_i} - D).$$
(3)

The MWS equilibrium contract parameters $(q^{\beta_L}, q^{\beta_H}, p^{\beta_L}, p^{\beta_H})$ result from the following maximization problem:

$$\max_{q^{\beta_L}, q^{\beta_H}, p^{\beta_L}, p^{\beta_H}} V_L(\beta_L) \tag{4}$$

s.t.
$$V_H(\beta_H) \ge V_H(\beta_L)$$
 (5)

$$\lambda(p^{\beta_H} - \pi_H q^{\beta_H}) + (1 - \lambda) \cdot (p^{\beta_L} - \pi_L q^{\beta_L}) \ge 0 \tag{6}$$

$$V_H(\beta_H) \ge V_H(\alpha_H). \tag{7}$$

The expected utility of low-risk individuals is maximized under the incentive compatibility constraint (5). The aggregate break-even constraint (6) displays the crucial difference to the RS framework, in which insurers break even individually on each contract. Constraint (7) ensures that there is no cross-subsidization from high risks to low risks. If this constraint is binding, $\overline{q^{\alpha_L} < D}$ for high-risk individuals not to be attracted by the insurance contract designed for low risks. the MWS contracts correspond to the RS contracts. If constraint (7) does not bind, high risks receive full coverage and are cross-subsidized by low risks.¹¹ Netzer and Scheuer (2010) show that, under the assumption of standard preferences, such cross-subsidization takes place if the fraction of high-risk individuals in the market λ falls short of a critical fraction λ_{RS} .¹² The assumption that the market is described by cross-subsidizing MWS contracts if $\lambda < \lambda_{RS}$ and by a non-crosssubsidizing RS separating equilibrium if $\lambda \geq \lambda_{RS}$ ensures a second-best efficient market allocation given the adverse selection externalities, as shown by Crocker and Snow (1985).¹³ If $\lambda < \lambda_{RS}$, the MWS equilibrium contracts (β_H , β_L) entail a cross-subsidy from low-risk individuals to high-risk individuals and high risks receive full coverage, i.e., $\beta_H = (p^{\beta_H}, D)$ and $\beta_L = (p^{\beta_L}, q^{\beta_L})$. The high-risk individuals' utility of a cross-subsidizing MWS contract, β_H , is then given by:

$$V_H(\beta_H) = u(w_0 - p^{\beta_H}).$$
 (8)

¹¹See e.g., Netzer and Scheuer (2010) or Mimra and Wambach (2014). RS contracts result as a special case of MWS contracts without cross-subsidization. We will separate those cases in order to highlight the different implications for our analysis. Therefore, throughout this chapter, we refer to MWS contracts if the statements made hold for both, the non-cross-subsidizing and the subsidizing case, to RS contracts for the non-cross-subsidizing MWS contracts and to cross-subsidizing MWS contracts for MWS contracts with non-zero cross-subsidization.

¹²We follow Crocker and Snow (1985) and Browne and Kamiya (2012) in the use of the notation λ_{RS} as the critical fraction of high risks in the market. However, those authors use it in the context of the transition from a RS separating equilibrium to a Wilson pooling equilibrium.

¹³Compare also Crocker and Snow (2008).



Figure 1: MWS Equilibrium

Figure 1 illustrates the MWS equilibrium. The individual's wealth state in the case of no loss, w_1 , is represented on the x-axis, whereas the wealth state in the case of a loss, w_2 , is displayed on the y-axis. The w_2^L -line and the w_2^H -line illustrate the insurer's break-even constraints for non-crosssubsidizing RS contracts for low risks and high risks, respectively. w_2^M illustrates the break-even line for the insurer if it were to offer the same contract to both high and low risks. The dotted curve that runs from the RS low risk contract, α_L , to the certainty line represents all feasible low risk contracts that satisfy the MWS constraints. It is therefore constraint by the insurer's non-crosssubsidizing and cross-subsidizing break-even lines for low risks. Along this dotted curve, a low risk individual's expected utility is maximized at the cross-subsidizing contract, β_L . The corresponding high risk cross-subsidized MWS contract, β_H , is located where the certainty line crosses the high risk person's indifference curve, U_H , that yields the same expected utility for the high risk individual as the low risk MWS contract, β_L . If the fraction of high risks λ in the market increases sufficiently for the green dotted MWS-curve to run entirely below the low risks' indifference curve U_L at the low risk RS contract α_L , any cross-subsidizing contract offers less expected utility to the low risks than their non-cross-subsidizing RS contract and the market is described by a non-cross-subsidizing RS separating equilibrium. In the cross-subsidizing MWS equilibrium, high risks always receive full coverage. In comparison to the non-cross-subsidized RS contract, α_H , they move to the north/east of the certainty line and obtain full coverage for a lower premium. The premium subsidy thereby is financed by the low risks, who pay an actuarially unfair premium in order to receive more coverage.

3.3 Policies with Screening Options

Similar to Browne and Kamiya (2012), we introduce a screening contract that offers full coverage in exchange for the fair premium, if individuals are willing to share a sufficient amount of information to reveal their true risk type.¹⁴ That information can, for instance, be provided through use of telemonitoring technologies. The screening contract is described by $\gamma_i = (p^{\gamma_i}, D)$ with $i \in \{H, L\}$. In order to clearly identify the effects of privacy concerns, we assume zero transaction costs for information retrieval. Therefore, costs for the implementation and maintenance of a telemonitoring device are not a decision criterion.¹⁵ The premium for a contract γ_i offered to individuals that have revealed their risk type *i* by sharing private information is given by:

$$p^{\gamma_i} = \pi_i D. \tag{9}$$

The contract with a screening option offers full coverage at a fair premium and therefore increases low risks' monetary utility, in comparison to either one of the contracts with partial coverage discussed in the previous subsection. We assume that policyholders' utility from insurance is not only determined by their monetary wealth, but it also takes into account the individuals' valuation of privacy and the resulting disutility from the level of screening agreed upon at contract inception.

Definition 1: The disutility resulting from sharing private information for individual j is described by $\psi_j \in (0, \infty)$.

Individuals decide whether or not to purchase an insurance product that requires revealing private information by trading off the maximization of expected utility of monetary wealth $U_i(w_i)$ against the minimization of disutility from sharing private information. Throughout this chapter,

¹⁴We assume that the revelation of private information allows for perfect classification in order to be able to clearly identify the effects.

¹⁵A brief discussion of this assumption is made in Section 7.

we refer to the latter as *privacy concerns*. It is modeled additively as a second attribute to the utility function.¹⁶ Hence, the utility of an individual with a contract $\gamma_i, i \in \{H, L\}$ is given by:

$$V_{i,\psi_{j}}(\gamma_{i}) = U_{i}(w_{i}) - \psi_{j}$$

$$= [(1 - \pi_{i}) \cdot u(w_{0} - p^{\gamma_{i}}) + \pi_{i} \cdot u(w_{0} - p^{\gamma_{i}} + D - D)] - \psi_{j}$$

$$= u(w_{0} - p^{\gamma_{i}}) - \psi_{j}$$

$$= u(w_{0} - \pi_{i}D) - \psi_{j}.$$
(10)

4 Equilibrium Analyses

4.1 Consumers' Participation Constraints

In order to specify the demand for screening policies γ_i with i = H, L, we consider cases in which individuals' utility from the screening contract is higher than their expected utility from an alternative contract offered to them. With $\psi_j > 0$, high risks will never have an incentive to choose the screening contract γ_H and will therefore never reveal their private information, regardless of their degree of aversion. For low-risk individuals, we need to differentiate between the underlying market equilibria, that is, whether the contract offered to them as an alternative to the screening contract is a non-cross-subsidizing RS separating equilibrium contract α_L or a cross-subsidizing MWS contract β_L .

Consider first the case in which a non-cross-subsidizing RS separating equilibrium exists in the market. In this case, as previously discussed, $\lambda \geq \lambda_{RS}$. Low risks will prefer a screening contract

¹⁶The multiattribute value function is given by the sum of two utility functions with different arguments. See, for instance, Eisenführ et al. (2010) and Keeney and Raiffa (1993). Numerous articles on insurance market equilibria have taken into account different types of consumers' characteristics and have modeled them as a second attribute to the consumers' utility function. This strand of literature considers characteristics, such as patience (e.g., Sonnenholzner and Wambach (2009)), overconfidence (e.g., Huang et al. (2010)), ambiguity aversion (e.g., Koufopoulos and Kozhan (2016)), and regret (e.g., Huang et al. (2016)). In the context of the valuation of privacy, this approach is taken by e.g., Filipova-Neumann and Welzel (2010).

over a RS contract, if and only if

$$V_{L,\psi_{j}}(\gamma_{L}) > V_{L}(\alpha_{L})$$

$$\Leftrightarrow u(w_{0} - \pi_{L}D) - \psi_{j} > (1 - \pi_{L}) \cdot u(w_{0} - \pi_{L}q^{\alpha_{L}}) + \pi_{L} \cdot u(w_{0} - \pi_{L}q^{\alpha_{L}} + q^{\alpha_{L}} - D)$$

$$\Leftrightarrow u(w_{0} - \pi_{L}D) - \psi_{j} > u(w_{0} - \pi_{L}D - \mu_{L}^{\alpha_{L}})$$

$$\Leftrightarrow \psi_{j} < u(w_{0} - \pi_{L}D) - u(w_{0} - \pi_{L}D - \mu_{L}^{\alpha_{L}}),$$
(11)

where $\mu_L^{\alpha_L}$ is the low risk's risk premium associated with the non-cross-subsidizing RS separating contract α_L .¹⁷ The interpretation of Inequality (11) is straightforward. For an individual to choose the insurance contract with screening, the extra utility gained from full insurance must exceed the disutility from revealing private information.

Consider next the case in which a cross-subsidizing MWS equilibrium exists in the market. In this case, $\lambda < \lambda_{RS}$. Now, the low risks' participation constraint for the screening contract is given by:

$$V_{L,\psi_{j}}(\gamma_{L}) > V_{L}(\beta_{L})$$

$$\Leftrightarrow u(w_{0} - \pi_{L}D) - \psi_{j} > (1 - \pi_{L}) \cdot u(w_{0} - p^{\beta_{L}}) + \pi_{L} \cdot u(w_{0} - p^{\beta_{L}} + q^{\beta_{L}} - D)$$

$$\Leftrightarrow u(w_{0} - \pi_{L}D) - \psi_{j} > u(w_{0} - p^{\beta_{L}} - \pi_{L}D + \pi_{L}q^{\beta_{L}} - \mu_{L}^{\beta_{L}})$$

$$\Leftrightarrow \psi_{j} < u(w_{0} - \pi_{L}D) - u(w_{0} - p^{\beta_{L}} - \pi_{L}D + \pi_{L}q^{\beta_{L}} - \mu_{L}^{\beta_{L}}),$$
(12)

where $\mu_L^{\beta_L}$ is the low risks' risk premium associated with the cross-subsidizing MWS contract for low risks β_L . The extra utility gained from full insurance and subsidizing the high risks must exceed the disutility from revealing private information.

If conditions (11) or (12) are fulfilled in the respective underlying market situation, low-risk individuals reveal their risk type in order to purchase the insurance product γ_L . This leads to symmetric information between those consumers and insurers. In other words, those low risks drop out of the pool of risks unknown to the insurer and receive full coverage at a fair premium.

 $^{1^{17}\}mu_L^{\alpha_L}$ represents the amount a low-risk policyholder would be willing to pay in addition to the fair insurance premium to obtain full insurance coverage in the absence of privacy concerns.

4.2 Privacy Concerns among Consumers

Definition 2: Let $F_L(\psi) \in [0,1]$ be the Cumulative Distribution Function (CDF) of a continuous distribution of privacy concerns $f_L(\psi)$ among low-risk individuals. An equilibrium is effectively a cut-off $\overline{\psi}_{\tau}$ with $\tau \in \{\alpha, \beta\}$ and a set of contracts (α_L, α_H) or (β_L, β_H) , respectively, such that:

- 1. (α_L, α_H) is a standard non-cross-subsidizing RS equilibrium given the fraction of high risks $\frac{\lambda}{(1-\lambda)(1-F_L(\bar{\psi}_{\alpha}))+\lambda}$ or (β_L, β_H) is a standard cross-subsidizing MWS equilibrium given the fraction of high risks $\frac{\lambda}{(1-\lambda)(1-F_L(\bar{\psi}_{\beta}))+\lambda}$, respectively.
- 2. $u(w_0 \pi_L D) V_L(\alpha_L) = \overline{\psi}_{\alpha}$ or $u(w_0 \pi_L D) V_L(\beta_L) = \overline{\psi}_{\beta}$, respectively.

That is, in equilibrium, cut-off types are indifferent between the (cross-subsidizing or non-crosssubsidizing) MWS equilibrium contract for low risks and the screening contract. Then all high-risk individuals stay in the market and the MWS equilibrium involves a lower fraction of low risks.¹⁸

Lemma 1: The resulting fraction of low risks in the new pool of risks unknown to the insurer is given by:

$$(1 - \lambda_{\tau}) \coloneqq \frac{(1 - \lambda)(1 - F_L(\overline{\psi}_{\tau}))}{(1 - \lambda)(1 - F_L(\overline{\psi}_{\tau})) + \lambda} < (1 - \lambda).$$

$$(13)$$

Consequently, the fraction of high risks in the new pool is given by:

$$\lambda_{\tau} \coloneqq \frac{\lambda}{(1-\lambda)(1-F_L(\overline{\psi}_{\tau}))+\lambda} > \lambda \tag{14}$$

with $\tau \in \{\alpha, \beta\}$.

Proof: See Appendix A.1.1.

In order to investigate how the option to reveal private information before contract inception affects market equilibria, we again must differentiate the two possible cases of the underlying market composition and the resulting market equilibria without the screening contract. In other words,

¹⁸In our subsequent analysis, we neglect the polar cases $F_L(\overline{\psi}_{\tau}) \in \{0, 1\}$. For $F_L(\overline{\psi}_{\tau}) = 0$, no individual is willing to reveal private information and the availability of a screening contract to the market has no effect on market equilibria as it does not attract any individuals. If $F_L(\overline{\psi}_{\tau}) = 1$, all low risk individuals are willing to share their private information in order to get full insurance at a fair premium. This case leads to a first-best market equilibrium with symmetric information.

we distinguish market equilibria with the screening option for a market that would result in a RS or a cross-subsidizing MWS equilibrium, respectively, had there not been the option to reveal consumers' risk types. The availability of a screening contract can alter the nature of the market equilibrium or the equilibrium configuration by increasing the fraction λ of high risks in the market with asymmetric information. As a result, we have to distinguish three mutually exclusive and collectively exhaustive cases.¹⁹

4.3 Persistence of a Non-cross-subsidizing RS Equilibrium

Proposition 1: Suppose it is the case that $\lambda_{RS} \leq \lambda$, i.e., without the screening option there is a non-cross-subsidizing RS equilibrium (α_H, α_L) . Then it holds that $\lambda_{RS} \leq \lambda < \lambda_{\alpha}$ and a noncross-subsidizing equilibrium persists. But the low risks with low privacy concerns (the ones whose utility from full insurance outweighs the disutility from screening) choose the screening contract with full insurance over the contract with partial coverage. Three contracts persist in equilibrium: $(\alpha_H, \alpha_L, \gamma_L)$.

Proof: The proof follows immediately from Lemma 1 that implies $\lambda < \lambda_{\alpha}$ and from the definition

of λ_{RS} .

¹⁹A similar distinction in a different context is made by Crocker and Snow (2008) who analyze the effect of background risk on the performance of insurance markets. In their framework, the existence of background risks increases the critical fraction λ_{RS} of high risks that alters the nature of the equilibrium rather than the fraction λ of high risks in the market.



Figure 2: Persistence of a Non-Cross-Subsidizing RS Equilibrium

Figure 2 illustrates the case in which a non-cross-subsidizing RS separating equilibrium exists, i.e., $\lambda_{RS} \leq \lambda$. Any cross-subsidizing MWS contract, $\beta_L = (q^{\beta_L}, p^{\beta_L})$, offers less expected utility to low risks than contract α_L since the dotted curve illustrating the set of potential cross-subsidizing MWS contracts for low risks runs entirely below the low risks' indifference curve U_L . The fraction of high risks in the market is already sufficiently high for a non-cross-subsidizing separating equilibrium (α_H, α_L) to exist and the availability of the screening contract can only increase the fraction of high risks in the pool of risks unknown to the insurer, i.e., it can only shift the green dotted curve of feasible cross-subsidizing contracts farther below the low risks' indifference curve U_L .

The new market equilibrium is described by three contracts, namely the screening contract γ_L and the two contracts α_H and α_L that persist in equilibrium and separate the high risks from low risks with high privacy concerns. In this case, the availability of a screening contract improves market performance by enabling low risks with low privacy concerns to gain full coverage without changing the other equilibrium contracts.

4.4 Evolution of a Cross-Subsidizing MWS Equilibrium

If $\lambda < \lambda_{RS}$ - that is, without screening, there is a cross-subsidizing MWS equilibrium (β_H, β_L) - then the availability of the screening contract can result in two possible scenarios depending on the fraction of individuals privacy concerns in the market:

Proposition 2: If the number of individuals that do not wish to share their private information is sufficiently high, such that $\lambda < \lambda_{\beta} < \lambda_{RS}$, the market equilibrium $(\beta'_H, \beta'_L, \gamma_L)$ is described by two cross-subsidizing MWS contracts and a contract offering the screening option.

Proof: The proof follows immediately from Lemma 1 that implies $\lambda < \lambda_{\beta}$ and from the definition of λ_{RS} .



Figure 3: Persistence of a Cross-Subsidizing MWS Equilibrium

The persistence of the cross-subsidizing MWS Equilibrium is illustrated in Figure 3. It shows that the insurer's pooled zero profit line shifts downwards (from w_2^M to $w_2^{M'}$) due to a higher fraction of high risks in the market. With a downward shifting joint zero profit line, all feasible combinations of cross-subsidizing MWS contract menus shift downwards as well. Since there is still a sufficient fraction of low risks in the market, not all feasible cross-subsidizing contracts for low risks (illustrated by the dotted curve) shift entirely below the low risks' indifference curve U_L . Therefore, a cross-subsidizing MWS contract β'_L will attract low-risk individuals with privacy concerns. A new equilibrium $(\beta'_L, \beta'_H, \gamma_L)$ is established. For the individuals who do not reveal their private information, a higher premium is associated with any given level of coverage, and lower coverage is granted for any given premium. Therefore, in the new equilibrium, high risks pay a higher premium for full coverage, low risks with privacy concerns receive less coverage and cross-subsidization is lower. The overall effect on market performance is ambiguous, as insurance coverage for low risks with high privacy concerns decreases while low risks with low privacy concerns receive full coverage.

Proposition 3: If the number of individuals that are willing to share their private information is sufficiently high, such that $\lambda < \lambda_{RS} < \lambda_{\beta}$, the market equilibrium no longer involves cross-subsidies and is now described by a three-contract separating equilibrium ($\alpha_H, \alpha_L, \gamma_L$).

Proof: The proof follows immediately from Lemma 1 that implies $\lambda < \lambda_{\beta}$ and from the definition of λ_{RS} .

Figure 4 shows (1) a cross-subsidizing MWS equilibrium (β_L, β_H) when no screening contract is offered, and (2) a non-cross-subsidizing RS separating equilibrium $(\alpha_H, \alpha_L, \gamma_L)$ when consumers can choose to purchase a fairly priced insurance policy conditional on revealing private information. The insurer's zero profit line that pools high risks and low risks with high privacy concerns shifts far below the low risks' indifference curve when the screening contract is introduced into the market. As a consequence, there is no cross-subsidizing MWS contract on the dotted curve that can attract low-risk individuals, so the market equilibrium is described by non-cross-subsidizing separating contracts in the RS sense. High risks have to pay a higher premium for full coverage and low risks with high privacy concerns receive less coverage. As in the previous case, the overall effect on market performance is ambiguous because low risks with low privacy concerns receive full coverage. The availability of a screening contract in this case eliminates any cross-subsidies.



Figure 4: From Cross-Subsidizing MWS to Non-Cross-Subsidizing RS Equilibrium

4.5 Efficiency Analysis

This subsection summarizes changes in market performance that result from the availability of a screening contract. To this end, Table 1 shows the changes in the equilibrium contract parameters for the respective consumer groups and the respective underlying market composition, where \uparrow indicates an increase in the respective contract parameter, \downarrow indicates a decrease, and \rightarrow indicates that the parameter remains unchanged.

Cases	High Risks	Low Risks with High	Low Risks with Low
		Privacy Concerns	Privacy Concerns
$\lambda_{RS} < \lambda \le \lambda_{\alpha}$ $(\alpha_H, \alpha_L) \to (\alpha_H, \alpha_L, \gamma_L)$	$q \rightarrow$	$q \rightarrow$	$q\uparrow$
$(\alpha_H, \alpha_L) \to (\alpha_H, \alpha_L, \gamma_L)$	$p \rightarrow$	$p \rightarrow$	$p\uparrow$
$\lambda \leq \lambda_{eta} < \lambda_{RS}$	$q \rightarrow$	$q\downarrow$	$q\uparrow$
$\lambda \leq \lambda_{\beta} < \lambda_{RS}$ $(\beta_H, \beta_L) \to (\beta'_H, \beta'_L, \gamma_L)$	$p\uparrow$	$p\downarrow$	$p\uparrow$
$\lambda < \lambda_{RS} \le \lambda_{\beta}$ $(\beta_H, \beta_L) \to (\alpha_H, \alpha_L, \gamma_L)$	$q \rightarrow$	$q\downarrow$	$q\uparrow$
$(\beta_H, \beta_L) \to (\alpha_H, \alpha_L, \gamma_L)$	$p\uparrow$	$p\downarrow$	$p\uparrow$

Table 1: Difference in Contract Parameters due to the Availability of a Screening Contract

In the case of a persistent RS equilibrium, low risks with low privacy concerns receive full coverage, while nothing changes for the two other consumer groups. Hence, if $\lambda_{RS} \leq \lambda < \lambda_{\alpha}$, the availability of a screening contract improves market efficiency.

If the market equilibrium in the absence of a screening contract is described by cross-subsidizing MWS contracts, i.e., $\lambda < \lambda_{RS}$, the equilibrium coverage for low risks with high privacy concerns decreases.²⁰ Low risks with less privacy concerns receive full coverage and the full coverage for high risks is unaffected. The effect on market performance is ambiguous. In this case, the equilibrium premium for high risks increases while low risks with high privacy concerns pay less premium and therefore reduce cross-subsidies. Hence, if at least one consumer values their privacy sufficiently high to prevent him or her from choosing the screening contract, the existence of such a contract decreases the likelihood of cross-subsidization.

5 Welfare Effects of a Screening Contract

5.1 Differences in Consumers' Expected Utility for the Respective Market Equilibria

For the respective scenarios analyzed in Section 4, we look at how the availability of a screening contract changes each consumer groups' expected utility (high risks, low risks with high privacy

 $^{^{20}}$ This corresponds to the second and the third row in Table 1.

concerns, and low risks with low privacy concerns) as well as utilitarian social welfare.²¹ In the subsequent welfare analysis, the second-best efficiency characteristic of the MWS contracts ensures that there is no possibility of improving the market's performance.²² We denote the expected utility of individuals who do not value their privacy sufficiently high to violate Inequality (11) and Inequality (12) and therefore choose the screening contract with $V_{L,\underline{\psi}_{\alpha}}$ and $V_{L,\underline{\psi}_{\beta}}$, respectively.²³ The impact of the availability of a screening contract on utilitarian social welfare is defined as:

$$\Delta_{\gamma}V \coloneqq \lambda \Delta_{\gamma}V_{H} + (1-\lambda)(1 - F_{L}(\overline{\psi}_{\tau}))\Delta_{\gamma}V_{L,\overline{\psi}_{\tau}} + (1-\lambda)F_{L}(\overline{\psi}_{\tau})\Delta_{\gamma}V_{L,\underline{\psi}_{\tau}}, \ \tau \in \{\alpha,\beta\}.$$
(15)

 $^{^{21}}$ There are several possible approaches to measure the welfare in insurance markets (see for example Hendren (2018)). To measure the changes in consumers' expected utility, there is no perfect baseline to compare to consumers' expected utility and utilitarian social welfare in the case of insurance market equilibria including a screening contract. A different approach would for instance be to choose the case of no insurance as a baseline. We choose the case of insurance market equilibria without screening options as a baseline in order to analyze how the availability of such a screening contract changes consumers' expected utility.

²²See Crocker and Snow (2008).

²³The use of the subscripts α and β should stress that the cut-off level of privacy concerns for low risks differs for a market with a non-cross-subsidizing RS equilibrium (denoted by α) and a market with a cross-subsidizing MWS equilibrium (denoted by β).

Proposition 4: The effect that the availability of a screening policy has on consumers' expected utility depends on the proportions of high risks, low risks with low privacy concerns, and low risks with high privacy concerns in the market, and is given by:

$$\Delta_{\gamma} V_{H} \coloneqq \begin{cases} V_{H}(\alpha_{H}) - V_{H}(\alpha_{H}) = 0, & \text{if } \lambda_{RS} \leq \lambda < \lambda_{\alpha} \\ V_{H}(\beta'_{H}) - V_{H}(\beta_{H}) < 0, & \text{if } \lambda < \lambda_{\beta} < \lambda_{RS} \\ V_{H}(\alpha_{H}) - V_{H}(\beta_{H}) < 0, & \text{if } \lambda < \lambda_{RS} < \lambda_{\beta} \end{cases}$$
(16)

for high-risk individuals,

$$\Delta_{\gamma} V_{L,\overline{\psi}_{\beta}} := \begin{cases} V_{L}(\alpha_{L}) - V_{L}(\alpha_{L}) = 0, & \text{if } \lambda_{RS} \leq \lambda < \lambda_{\alpha} \\ V_{L}(\beta'_{L}) - V_{L}(\beta_{L}) < 0, & \text{if } \lambda < \lambda_{\beta} < \lambda_{RS} \\ V_{L}(\alpha_{L}) - V_{L}(\beta_{L}) < 0, & \text{if } \lambda < \lambda_{RS} < \lambda_{\beta} \end{cases}$$
(17)

for low-risk individuals who value their privacy, and

$$\Delta_{\gamma} V_{L,\underline{\psi}_{\beta}} \coloneqq \begin{cases} V_{L,\underline{\psi}_{\beta}}(\gamma_{L}) - V_{L}(\alpha_{L}) > 0, & \text{if } \lambda_{RS} \le \lambda < \lambda_{\alpha} \\ V_{L,\underline{\psi}_{\beta}}(\gamma_{L}) - V_{L}(\beta_{L}) > 0, & \text{if } \lambda < \lambda_{\beta} < \lambda_{RS} \\ V_{L,\underline{\psi}_{\beta}}(\gamma_{L}) - V_{L}(\beta_{L}) > 0, & \text{if } \lambda < \lambda_{RS} < \lambda_{\beta} \end{cases}$$
(18)

for low-risk individuals who do not value their privacy sufficiently to reject the screening contract. The resulting difference in social welfare is given by:

$$\Delta_{\gamma}V \coloneqq \begin{cases} V(\alpha_{H}, \alpha_{L}, \gamma_{L}) - V(\alpha_{H}, \alpha_{L}) > 0, & \text{if } \lambda_{RS} <\leq \lambda < \lambda_{\alpha} \\ V(\beta_{L}', \beta_{H}', \gamma_{L}) - V(\beta_{L}, \beta_{H}) <> 0, & \text{if } \lambda < \lambda_{\beta} < \lambda_{RS} \\ V(\alpha_{H}, \alpha_{L}, \gamma_{L}) - V(\beta_{H}, \beta_{L}) <> 0, & \text{if } \lambda < \lambda_{RS} < \lambda_{\beta}. \end{cases}$$
(19)

Proof: See Appendix A.2.1.

Since, for $\lambda_{RS} \leq \lambda < \lambda_{\alpha}$, high-risk individuals and low-risk individuals with high privacy concerns choose the non-cross-subsidizing RS contract as in the case without a screening option,

their expected utility does not change with the availability of a contract that requires screening and offers full insurance at a fair premium. For the case $\lambda < \lambda_{RS}$, i.e., when the fraction of high risks in the market implies a market equilibrium without a screening contract that involves cross-subsidies from low risks to high risks, the availability of the screening contract γ_L leads to a utility shift from high risks and low risks with high privacy concerns to low risks with low privacy concerns.

The impact of the availability of a screening contract on utilitarian social welfare depends, among other factors, on the proportions of high risks, low risks with low privacy concerns and low risks with high privacy concerns in the market. For $\lambda_{RS} \leq \lambda < \lambda_{\alpha}$, the availability of a screening contract γ_L leads to a Pareto improvement of utilitarian social welfare. The welfare gain equals the aggregate expected utility gain of low risks with low privacy concerns who receive full insurance at a fair premium rather than partial coverage. For $\lambda \leq \lambda_{\beta} < \lambda_{RS}$ and $\lambda < \lambda_{RS} < \lambda_{\beta}$, the overall change in utilitarian social welfare resulting from the availability of a screening contract γ_L is ambiguous. The option to reveal private information in this context leads to a welfare gain if the aggregate increase in expected utility for low risks with low privacy concerns outweighs the aggregate expected utility loss for high risks and low risks with high privacy concerns.

5.2 Illustration of Differences in Consumers' Expected Utility

In the following, we illustrate how the underlying market composition affects the expected utility of different consumer types when a screening contract is offered. For the graphical illustration, we concentrate on the two polar cases of privacy concerns.²⁴ We assume that either individuals do not exhibit any privacy concerns, or they value their privacy sufficiently high to violate Inequality (11) or Inequality (12), respectively, i.e., $\psi_j \in \{0, \overline{\psi}_{\tau}\}$ with $\tau \in \{\alpha, \beta\}$; $V_{L,\overline{\psi}_{\alpha}}(\gamma_L) < V_L(\alpha_L)$ and $V_{L,\overline{\psi}_{\beta}}(\gamma_L) < V_L(\beta_L)$. Hence, individuals with privacy concerns choose not to reveal their private information, since the disutility resulting from doing so outweighs the utility gain from full insurance coverage. Individuals who do not exhibit privacy concerns choose to reveal their private information and will not suffer any loss of utility as a result. Therefore, the utility illustrated in this subsection corresponds to consumers' monetary utility. We denote the respective fraction of individuals who value their privacy sufficiently high with k_{τ} , where $\tau \in \{\alpha, \beta\}$.

 $^{^{24}}$ This assumption allows a two-dimensional illustration and therefore a clearer interpretation of the resulting welfare effects.

The heat diagrams presented below show the fraction of high-risk individuals in the market on the x-axis and the fraction of individuals with privacy concerns among low risks on the y-axis. The expected utility change for the respective consumer group is displayed by different colors with the respective values measured by the bar on the right of each diagram. We choose example values for individuals' utility function, their loss probability, their initial wealth, and the loss they face: $u(w) = ln(w), \pi_H = 0.7, \pi_L = 0.4, w_0 = 10, D = 9.^{25}$ For those values, the critical fraction of high risks is given by $\lambda_{RS} \approx 0.58.^{26}$



Figure 5: High Risks' Difference in Expected Monetary Utility due to the Availability of a Screening Contract

For any values $\lambda < \lambda_{RS} \approx 0.58$, the insurance market equilibrium is described by crosssubsidizing MWS contracts, if no screening contract is offered. Figure 5 shows that the utility change for high risks in this case heavily depends on how the market composition changes with the availability of such a contract. If the fraction of low risks with privacy concerns is sufficiently high for the new market equilibrium to still be described by cross-subsidizing MWS contracts (as shown in the light red-shaded area on the left of the diagram), the loss in utility for high risks is lower

²⁵We chose the loss probabilities in accordance with our graphical illustrations for the equilibrium analysis. Qualitatively similar results are obtained using lower values. For the calculation of the optimal coverage for the RS low risk contract α_L , see Appendix A.3.

²⁶The critical fraction of high risk λ_{RS} is implicitly determined by $V_L(\alpha_L) = V_L(\beta_L) \Leftrightarrow (1 - \pi_L) \cdot u(w_0 - p^{\beta_L}) + \pi_L \cdot u(w_0 - p^{\beta_L} + q^{\beta_L} - D)$, where p^{β_L} and q^{β_L} depend on λ .

than if a non-cross-subsidizing equilibrium results with the availability of the screening contract (as shown in the darker red shaded area). High risks face the highest loss of utility when their share in the market is very low, but the availability of the screening contract still leads to a non-crosssubsidizing RS equilibrium due to the very low fraction of low risks with privacy concerns. This is due to the fact that, with very few high risks in the market, low risks are willing to subsidize high risks to a large extent, in order to gain more coverage. Therefore, the premium for high risks in this case is very low and the reference level of utility in the absence of a screening contract is high.



Figure 6: Difference in Expected Monetary Utility for Low Risks with Privacy Concerns due to the Availability of a Screening Contract

Figure 6 shows the difference in expected utility for low risks with privacy concerns resulting from the availability of a screening contract. Given that the market equilibrium is described by cross-subsidizing MWS contracts in the absence of the screening policy, the difference in expected utility for low risks with privacy concerns follows roughly the same pattern as the difference in high risks' utility. Yet, in comparison with the high risks' change in utility, the expected utility loss that low risks with privacy concerns face, when the availability of the screening contract leads to a non-cross-subsidizing RS equilibrium, is lower relative to the expected utility loss they suffer if a cross-subsidizing MWS equilibrium results.

Although their probability of loss does not change, low-risk individuals who do not share private

information suffer a loss of expected utility as a result of the screening contract being introduced to the market. This loss is highest when the initial fraction of high risks in the market and the fraction of low risks with privacy concerns are both relatively low. This case corresponds to a change from a cross-subsidizing MWS equilibrium with a low level of cross-subsidization to a non-cross-subsidizing RS equilibrium. It is illustrated by the dark red shaded area in the left corner at the bottom of the heat diagram in Figure 6. Since the availability of a screening contract in this case implies a change from a situation in which many low risks have to subsidize only a few high risks to a situation in which a few low risks have to subsidize many high risks, low risks with privacy concerns now choose to forgo coverage instead of paying expensive cross-subsidies.

The bright red shaded rectangle on the right side of both Figure 5 and Figure 6 shows that neither high-risk individuals nor low-risk individuals with privacy concerns face any change in expected utility if the market composition leads to a non-cross-subsidizing RS equilibrium in the absence of a screening contract, i.e., for $\lambda \geq \lambda_{RS} \approx 0.58$.



Figure 7: Difference in Expected Monetary Utility for Low Risks without Privacy Concerns due to the Availability of a Screening Contract

The obvious winners from the availability of the screening contract are low risks who experience no disutility from revealing their private information. Their expected utility gain (displayed in Figure 7) increases with the fraction of high risks in the market, as the reference level of expected utility in case of the non-existence of the screening contract decreases with the fraction of high-risk individuals. Their utility gain is highest when the market equilibrium is described by two noncross-subsidizing RS separating contracts in the absence of the screening policy (as displayed in the dark green shaded area). The screening contract enables them to obtain full insurance coverage, whereas the RS contract for low risks features a high deductible.

The impact of the availability of a screening contract on utilitarian social welfare is ambiguous, as it depends on the composition of individuals in the market with respect to their risk type and privacy concerns. Figure 8 illustrates the Pareto improvement of utilitarian social welfare resulting from the persistence of a non-cross-subsidizing RS separating equilibrium with the green shaded area on the right of the heat diagram. The highest welfare gain resulting from the availability of a screening contract is reached when there are just enough high-risk individuals in the market for a non-cross-subsidizing RS separating equilibrium to exist in the absence of the screening contract, and few low-risk individuals exhibit privacy concerns, i.e., the number of individuals who benefit from the availability of a screening contract is very high. This case is represented by the dark green shaded area. If a cross-subsidizing MWS equilibrium exists in the absence of a screening contract, the aggregate expected utility loss of high risks and of low-risk individuals with privacy concerns can outweigh the aggregate utility gain of low risks without privacy concerns. The welfare loss is highest when the market composition is such that the availability of the screening contract causes a change from a cross-subsidizing MWS equilibrium to a non-cross-subsidizing RS separating equilibrium, which is represented by the dark red in the heat diagram. The white curve that separates the red shaded area from the green shaded area illustrates the case when the utility gain from a screening contract for low risks without privacy concerns exactly outweight the utility loss of high risks and low risks with privacy concerns caused by a change from a cross-subsidizing equilibrium to a non-cross-subsidizing equilibrium.



Figure 8: Difference in Utilitarian Social Welfare due to the Introduction of a Screening Contract

6 Welfare Effects of Privacy Concerns

6.1 Differences in Consumers' Expected Utility for the Respective Market Equilibria

Thusfar, we have analyzed how the availability of a screening contract influences consumers' expected utility and overall social welfare, given that some consumers value their privacy. In this section, we examine the impact of such privacy concerns on consumers' expected utility and overall social welfare, given the availability of a screening contract. To achieve this, we consider consumers' expected utility resulting from insurance market equilibria that include a screening policy and compare this when some consumers value their privacy, versus when no consumer exhibits any privacy concerns. In the latter case, all consumers are offered full insurance coverage at a fair premium. The impact of privacy concerns on utilitarian social welfare is defined as:

$$\Delta_{\psi}V \coloneqq \lambda \Delta_{\psi}V_{H} + (1-\lambda)(1 - F_{L}(\overline{\psi}_{\tau}))\Delta_{\psi}V_{L,\overline{\psi}_{\tau}} + (1-\lambda)F_{L}(\overline{\psi}_{\tau})\Delta_{\psi}V_{L,\underline{\psi}_{\tau}}, \ \tau \in \{\alpha,\beta\}.$$
(20)

Proposition 5: The difference in consumers' expected utility resulting from insurance market equilibria when some consumers value their privacy versus when consumers do not exhibit privacy concerns depends on the proportions of high risks, low risks with low privacy concerns, and low risks with privacy concerns in the market. It is given by:

$$\Delta_{\psi} V_{H} \coloneqq \begin{cases} V_{H}(\alpha_{H}) - V_{H}(\alpha_{H}) = 0, & \text{if } \lambda_{RS} < \lambda_{\tau}, \tau \in \{\alpha, \beta\} \\ V_{H}(\beta'_{H}) - V_{H}(\alpha_{H}) > 0, & \text{if } \lambda_{\beta} < \lambda_{RS} \end{cases}$$
(21)

for high-risk individuals,

$$\Delta_{\psi} V_{L,\overline{\psi}_{\beta}} \coloneqq \begin{cases} V_L(\alpha_L) - V_L(\gamma_L) < 0, & \text{if } \lambda_{RS} < \lambda_{\tau}, \tau \in \{\alpha, \beta\} \\ V_L(\beta'_L) - V_L(\gamma_L) < 0, & \text{if } \lambda_{\beta} < \lambda_{RS} \end{cases}$$
(22)

for low-risk individuals who do exhibit privacy concerns in one of the cases, and

$$\Delta_{\psi} V_{L,\underline{\psi}_{\beta}} \coloneqq V_{L,\underline{\psi}_{\beta}}(\gamma_L) - V_L(\gamma_L) < 0 \tag{23}$$

for low-risk individuals who do not value their privacy sufficiently to reject the screening contract. The resulting difference in social welfare is given by:

$$\Delta_{\psi}V \coloneqq \begin{cases} V(\alpha_H, \alpha_L, \gamma_L) - V(\gamma_L, \alpha_H) < 0, & \text{if } \lambda_{RS} < \lambda_{\tau}, \tau \in \{\alpha, \beta\} \\ V(\beta'_L, \beta'_H, \gamma_L) - V(\gamma_L, \alpha_H) <> 0, & \text{if } \lambda_{\beta} < \lambda_{RS}. \end{cases}$$
(24)

Proof: See Appendix A.2.2.

Since, for $\lambda_{RS} < \lambda_{\tau}, \tau \in \{\alpha, \beta\}$, high risks as well as low risks with low privacy concerns receive the full coverage for the fair premium in both cases (i.e., the market equilibrium results from consumers with privacy concerns and the market equilibrium results from consumers with low privacy concerns), their loss of utility stems purely from non-monetary disutility, their privacy concerns. The utility loss for loss risks with high privacy concerns stems from their privacy concerns as well as the coverage differential in the two cases. When $\lambda < \lambda_{RS}$, that is, when the fraction of high risks in the market implies a market equilibrium without a screening contract that involves cross-subsidies from low risks to high risks, privacy concerns lead to a utility shift from low risks to high risks.

The difference in social welfare resulting from insurance market equilibria in the case in which some consumers value their privacy to the case in which consumers do not exhibit privacy concerns depends on the proportion of high risks, low risks with low privacy concerns, and low risks with high privacy concerns in the market. For $\lambda_{RS} < \lambda_{\tau}, \tau \in \{\alpha, \beta\}$, the existence of privacy concerns leads to a welfare loss. For $\lambda \leq \lambda_{\beta} < \lambda_{RS}$, the difference in social welfare, resulting from insurance market equilibria in the case in which some consumers value their privacy, and social welfare, resulting from insurance market equilibria in the case in which consumers do not exhibit any privacy concerns, is ambiguous. Privacy concerns lead to a welfare gain if the aggregate increase in high risks' expected utility outweighs the aggregate expected utility loss for low risks.

6.2 Illustration of Differences in Consumers' Expected Utility

Next, we illustrate the sensitivity of the difference in consumers' expected utility when some consumers value their privacy, versus when consumers have no privacy concerns with respect to the underlying market composition. For the graphical illustration, we rely on the differences in consumers' expected utility and, as in Section 5.2, we concentrate on the two polar cases of privacy concerns. We assume that individuals either exhibit no privacy concerns, or they value their privacy sufficiently high to violate Inequality (11) or Inequality (12), respectively. Therefore, we compare consumers' monetary expected utility resulting from insurance market equilibria that include a screening policy when some consumers value their privacy, versus when no consumer exhibits any privacy concerns. The heat diagrams show the fraction of high-risk individuals in the market on the x-axis and the fraction of individuals with privacy concerns among low risks on the y-axis. The expected utility change for the respective consumer group is displayed by different colors with the respective values measured by the bar on the right of each diagram. We choose the same values for the utility functions, loss probabilities, initial wealth, and losses faced as in Section 5.2.

For low risks without privacy concerns, there is no difference in expected utility resulting from insurance market equilibria when some consumers value their privacy, versus when consumers exhibit no privacy concerns. In both cases, they receive full coverage for a fair premium, independently of the fraction of high risks in the market.



Figure 9: Difference in High Risks' Expected Monetary Utility due to Privacy Concerns

The same holds for high-risk individuals, although only when the fraction of high risks in the market is sufficiently high for a non-cross-subsidizing RS equilibrium to exist. This is illustrated by the light green shaded area in Figure 9. If the fraction of high risks in the market is low and some low risks value their privacy, high risks benefit from these privacy concerns through cross-subsidization, as shown in the dark green area. The fewer high risks are in the market and the more low risks value their privacy, the higher is each high-risk individual's benefit from these privacy concerns. Hence, high-risk individuals cannot suffer a loss of monetary utility resulting from anyone's privacy concerns, but they can benefit from low risk's privacy concerns.

Figure 10 shows the difference in expected utility, when some consumers value their privacy, versus when consumers exhibit no privacy concerns, for those low risk individuals who do value their privacy in the first case. Note that this figure and all other figures in this section do not capture any disutility stemming directly from privacy concerns. All utility differences in this section describe differences in monetary utility that result indirectly from the existence of privacy concerns via the effect on the insurance market equilibrium. While high risks can only benefit from the existence of privacy can only privacy concerns in terms of expected monetary utility, low risks who value their privacy can only

suffer a loss of expected monetary utility therefrom. If the fraction of high risks in the market is low and some low risks value their privacy, cross-subsidization leads to a loss in expected monetary utility for those low risks. Their loss of utility is higher, however, if the market is characterized by a non-cross-subsidizing RS equilibrium, because low risks have to forgo a sufficient amount of insurance coverage to self-select from high risks while still protecting their privacy. Therefore, exhibiting privacy concerns harms low-risk individuals on a financial level.



Figure 10: Difference in Expected Monetary Utility for Low Risks with Privacy Concerns due to Privacy Concerns



Figure 11: Difference in Utilitarian Social Welfare due to the Existence of Privacy Concerns

7 Discussion

Thusfar, we have treated telemonitoring devices as costless, yet such costs could be paid in several different ways. For instance, if only consumers who wish to implement such telemonitoring devices have to pay for the occurring costs, this will result in a lower fraction of consumers electing such policies and a higher utility for high risks and low risks with privacy concerns. Conversely, if costs were shared equally among all policyholders, this effect would be small, as consumers can only influence the costs resulting from their own decisions. Another assumption maintained throughout is that screening allows for perfect classification. If screening were, however, noisy, i.e the information retrieved is insufficient to make an accurate prediction of individuals' risk type, high risks might have an incentive to purchase a screening contract as well. Further, in this setting, privacy concerns could be lower because information sharing is noisy and perfect identification is not possible.

Analyzing alternative frameworks might also help to understand how the effects change in different regulatory settings: For instance, one could think of a case where screening becomes a conditional requirement for the insurance contract to come into effect, for instance, if automobile producers were to pre-install monitoring devices in all vehicles. When full information sharing is enforced, information is symmetric and the insurer can price individuals according to their respective accident probabilities. This setting raises the question as to whether high-risk individuals are still insurable when they have to reveal their risk type. Further, in this case, there could be two possible scenarios. (1) If it is possible to not purchase insurance at all, e.g., by not buying a car, individuals with a high level of privacy concerns will choose to do so, and the market composition of risks depends on the correlation between the accident probability and privacy concerns. (2) If the individual must be insured, the enforced screening leads to a substantial welfare loss resulting from the disutility policyholders obtain by sharing private information.

Other articles have focused on multilevel heterogeneity in the context of asymmetric information in insurance markets. For instance, Wambach (2000) introduces unobservable wealth as a second level of heterogeneity across consumers, and he shows that wealth types are pooled while risk types are separated when wealth differences are sufficiently small. For large differences in wealth, however, partial risk pooling contracts are feasible. Smart (2000) analyzes insurance buyers that differ with respect to loss probability and their degree of risk aversion. He finds that firms cannot use deductibles to screen high risk individuals when differences in risk aversion are sufficiently large. Types are either pooled in equilibrium, or insurers can separate them by charging a loading above the actuarially fair premium. With respect to multilevel heterogeneity, our framework differs from these articles as we introduce a screening contract in equilibrium that is necessary to determine the effect of the second level of heterogeneity - privacy concerns. For some consumers, privacy concerns prevent insurers from observing the first level of heterogeneity - the loss probability - directly via this screening contract.

8 Conclusion

The current study considers an insurance market with asymmetric information consisting of risk neutral nonmyopic insurers that operate in a competitive market environment, and risk-averse consumers who differ in their risk type and valuation of privacy. We build on the framework developed by Wilson (1977), Miyazaki (1977), and Spence (1978) that yields the second best efficient separating, cross-subsidizing, jointly zero-profit making Miyazaki-Wilson-Spence (MWS) contracts.

We introduce the possibility that consumers can reveal their risk type for a certain subjective cost in exchange for a premium adjustment. We show analytically how this affects standard results regarding insurance market equilibria in the MWS framework, as well as consumers' individual expected utility and social welfare.

The MWS insurance market equilibrium outcomes depend on the fraction of high-risk individuals in the market. When this fraction exceeds a critical value, a cross-subsidizing contract cannot attract low-risk consumers and therefore the market equilibrium is described by two non-crosssubsidizing self-selecting separating contracts. Since a screening contract will only attract low-risk individuals, the fraction of high-risk individuals in the market with asymmetric information can only increase due to the availability of such a policy. As a result, the availability of a screening contract does not break up an existing non-cross-subsidizing RS separating equilibrium. We show that the availability of a screening contract can lead to a Pareto improvement of social welfare and a Pareto-superior market allocation if the fraction of high risks in the market without a screening contract is sufficiently high for the market equilibrium to be described by self-selection contracts in the Rothschild-Stiglitz sense. If a cross-subsidizing MWS equilibrium exists in the absence of a screening contract, however, the availability of such a policy decreases or even eliminates crosssubsidies. The resulting equilibrium depends on the fraction of low risks with high privacy concerns. The premium for an insurance policy that does not require policyholders to reveal private information then depends on the number of consumers choosing a screening contract. Given the prior existence of a cross-subsidizing MWS equilibrium, the availability of a screening contract results in less coverage for low risks with high, and high risks pay a higher premium for full coverage. Utility is shifted from individuals who do not reveal their private information to those who choose to reveal. In this case, the impact a screening contract has on the insurance market as well as on social welfare is ambiguous and depends on the composition of individuals in the market, with respect to their risk type and privacy concerns. The welfare loss is highest if the availability of the screening contract causes a change in the nature of the equilibrium, from a cross-subsidizing MWS equilibrium to a non-cross-subsidizing RS separating equilibrium. If at least one consumer values his or her privacy sufficiently high to prevent him or her from choosing the screening contract, the existence of such a contract decreases the likelihood of cross-subsidization.

We further examine how the existence of privacy concerns in insurance markets affects con-

sumers' utility and social welfare, by comparing them when some consumers value their privacy sufficiently highly to reject the offer of a screening contract, to the case in which no consumer values own privacy. If privacy concerns lead to cross subsidization, utility is shifted from the low risks, who exhibit these privacy concerns, to the high risks. Yet, the loss of utility for low risks with high privacy concerns stemming from privacy concerns is higher when the market equilibrium does not involve cross-subsidization, since the protection of private information in this case requires them to forgo a substantial amount of insurance coverage. With respect to monetary utility, privacy concerns harm the ones who exhibit them, and can improve the utility of high risks. Moreover, they can be welfare-enhancing, when there are few high risks in the pool and hardly any low risks are willing to share their information. In order to mitigate the disadvantages that low risks have from valuing their private information, possible approaches could be using a redistributive scheme, implementing stricter regulation with respect to data usage for pricing, or to try minimizing privacy concerns, e.g., with better data protection.

Our analysis provides a theoretical substantiation for the discussion of consumer protection in the context of digitalization. It shows that new technologies bring new ways to challenge crosssubsidization in insurance markets and stresses the negative externalities that digitalization can have on consumers who are unwilling to take part in this development. At the same time, our results support the need to protect and anonymize policyholder data in order to lower perceived costs of disclosure. Such consumer protection may also be achieved through digitalization, for instance by the use of block chain technology.

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A Appendix

A.1 Proofs for the Equilibrium Analysis

A.1.1 Proof of Lemma 1:

Given Definition 2, the fraction of individuals that reveal their information by choosing the screening contract and hence leave the pool of risks the insurer cannot identify is given by $(1 - \lambda)F_L(\overline{\psi}_{\tau})$. Therefore, the fraction of consumers who do not wish to reveal their information and therefore build a new pool of risks unknown to the insurer is described by $(1-\lambda)(1-F_L(\overline{\psi}_{\tau}))+\lambda$. \Box

A.2 Proofs for the Welfare Analysis

A.2.1 Proof of Proposition 4:

To show:

$$(i) \qquad \Delta_{\gamma} V_{H} \coloneqq \begin{cases} (i.i) & V_{H}(\alpha_{H}) - V_{H}(\alpha_{H}) = 0, & \text{if } \lambda_{RS} \leq \lambda < \lambda_{\alpha} \\ (i.ii) & V_{H}(\beta'_{H}) - V_{H}(\beta_{H}) < 0, & \text{if } \lambda < \lambda_{\beta} < \lambda_{RS} \\ (i.iii) & V_{H}(\alpha_{H}) - V_{H}(\beta_{H}) < 0, & \text{if } \lambda < \lambda_{RS} < \lambda_{\beta} \end{cases}$$

$$(ii) \qquad \Delta_{\gamma} V_{L,\overline{\psi}_{\beta}} \coloneqq \begin{cases} (ii.i) & V_L(\alpha_L) - V_L(\alpha_L) = 0, & \text{if } \lambda_{RS} \le \lambda < \lambda_{\alpha} \\ (ii.ii) & V_L(\beta'_L) - V_L(\beta_L) < 0, & \text{if } \lambda < \lambda_{\beta} < \lambda_{RS} \\ (ii.iii) & V_L(\alpha_L) - V_L(\beta_L) < 0, & \text{if } \lambda < \lambda_{RS} < \lambda_{\beta} \end{cases}$$

$$(iii) \qquad \Delta_{\gamma} V_{L,\underline{\psi}_{\beta}} \coloneqq \begin{cases} (iii.i) & V_{L,\underline{\psi}_{\beta}}(\gamma_L) - V_L(\alpha_L) > 0, & \text{if } \lambda_{RS} \le \lambda < \lambda_{\alpha} \\ (iii.ii) & V_{L,\underline{\psi}_{\beta}}(\gamma_L) - V_L(\beta_L) > 0, & \text{if } \lambda < \lambda_{\beta} < \lambda_{RS} \\ (iii.iii) & V_{L,\underline{\psi}_{\beta}}(\gamma_L) - V_L(\beta_L) > 0, & \text{if } \lambda < \lambda_{RS} < \lambda_{\beta} \end{cases}$$

$$(iv) \qquad \Delta_{\gamma}V := \begin{cases} (iv.i) & V(\alpha_{H}, \alpha_{L}, \gamma_{L}) - V(\alpha_{H}, \alpha_{L}) > 0, & \text{if } \lambda_{RS} \leq \lambda < \lambda_{\alpha} \\ (iv.ii) & V(\beta_{L}', \beta_{H}', \gamma_{L}) - V(\beta_{L}, \beta_{H}) <> 0, & \text{if } \lambda < \lambda_{\beta} < \lambda_{RS} \\ (iv.iii) & V(\alpha_{H}, \alpha_{L}, \gamma_{L}) - V(\beta_{H}, \beta_{L}) <> 0, & \text{if } \lambda < \lambda_{RS} < \lambda_{\beta} \end{cases}$$

(i):

(i.i): The proof is obvious and therefore omitted.

(i.ii):

$$V_H(\beta'_H) - V_H(\beta_H) < 0$$

$$\Leftrightarrow u(w_0 - p^{\beta'_H}) - u(w_0 - p^{\beta_H}) < 0$$

$$\Leftrightarrow u(w_0 - p^{\beta'_H}) < u(w_0 - p^{\beta_H})$$

Since the utility function is increasing in wealth, this holds due to $p^{\beta'_H} > p^{\beta_H}$.

(i.iii):

$$V_H(\alpha_H) - V_H(\beta_H) < 0$$

$$\Leftrightarrow u(w_0 - \pi_H D) - u(w_0 - p^{\beta_H}) < 0$$

$$\Leftrightarrow u(w_0 - \pi_H D) < u(w_0 - p^{\beta_H})$$

Since the utility function is increasing in wealth, this holds due to $\pi_H D > p^{\beta_H}$.

(ii):

(*ii.i*): The proof is obvious and therefore omitted.

(*ii.ii*): For any given level of coverage, low risks with high privacy concerns have to pay a higher

premium for a cross-subsidizing contract if the fraction of high risks in the market is higher. Hence, the maximum expected utility a low-risk individual can get from a cross-subsidizing contract based on a higher fraction of high risks is lower. MWS contracts are determined by maximizing low risks' expected utility within the set of feasible cross-subsidizing contracts that satisfy conditions (5), (6), and (7). Since the contract β' is determined to maximize low risks' expected utility based on a higher fraction of high risks than the contract β (see Lemma 1), it is

$$V_L(\beta'_L) - V_L(\beta_L) < 0$$

(*ii.iii*): If Constraint (7) is binding, the MWS contracts correspond to the RS contracts. Therefore, the low risk RS contract α_L lays within the set of feasible cross-subsidizing contracts that lowrisk individuals maximize their expected utility over. Hence, low risks expected utility $V_{L,\overline{\psi}_{\beta}}(\alpha_L)$ stemming from a contract α_L can never exceed their expected utility $V_L(\beta_L)$ from a cross-subsidizing MWS contract β_L and it is $V_{L,\overline{\psi}_{\beta}}(\alpha_L) = V_L(\beta_L) \Leftrightarrow \alpha_L = \beta_L$, i.e., if and only if low-risk individuals' expected utility of the two contracts is the same, the contracts are identical.

(iii):

(*iii.i*): Expected utility of a screening contract for low risks with low privacy concerns:

$$V_{L,\underline{\psi}_{\alpha}}(\gamma_L) = u(w_0 - \pi_L D) - \psi_j$$

(*iii.ii*) & (*iii.iii*): Since low-risk individuals choose the screening contract if and only if their privacy concerns are sufficiently low for the participation constraint (12) to hold, (*iii.ii*) and (*iii.iii*) hold by construction.

The change in individual expected utility for low-risk individuals with low privacy concerns is

$$\begin{aligned} \Delta_{\gamma} V_{L,\underline{\psi}_{\alpha}} &\coloneqq V_{L,\underline{\psi}_{\alpha}}(\gamma_L) - V_L(\alpha_L) \\ &= u(w_0 - \pi_L D) - \psi_j - \left[(1 - \pi_L) u(w_0 - \pi_L q^{\alpha_L}) + \pi_L u(w_0 - \pi_L q^{\alpha_L} + q^{\alpha_L} - D) \right] \\ &> 0 \end{aligned}$$

 $(iv) {:}$ We make use of the definition

$$\Delta_{\gamma}V \coloneqq \lambda \Delta_{\gamma}V_{H} + (1-\lambda)(1 - F_{L}(\overline{\psi}_{\tau}))\Delta_{\gamma}V_{L,\overline{\psi}_{\tau}} + (1-\lambda)F_{L}(\overline{\psi}_{\tau})\Delta_{\gamma}V_{L,\underline{\psi}_{\tau}}, \ \tau \in \{\alpha,\beta\}$$

(iv.i): Aggregate consumers' expected utility without a screening contract in a non-crosssubsidizing RS separating equilibrium:

$$V(\alpha_H, \alpha_L) = \lambda V_H(\alpha_H) + (1 - \lambda) V_L(\alpha_L)$$

Aggregate consumers' expected utility with a screening contract in a non-cross-subsidizing RS separating equilibrium:

$$V(\alpha_H, \alpha_L, \gamma_L) = \lambda V_H(\alpha_H) + (1 - \lambda)(1 - F_L(\overline{\psi}_{\alpha}))V_L(\alpha_L)$$
$$+ (1 - \lambda) \left[F_L(\overline{\psi}_{\alpha})u(w_0 - \pi_L D) - \int_0^{\overline{\psi}_{\alpha}} f_L(\psi)\psi d\psi \right]$$

$$\begin{split} \Delta V_{\gamma} &\coloneqq V(\alpha_{H}, \alpha_{L}, \gamma_{L}) - V(\alpha_{H}, \alpha_{L}) \\ &= \lambda \Delta_{\gamma} V_{H} + (1 - \lambda)(1 - F_{L}(\overline{\psi}_{\alpha}))\Delta_{\gamma} V_{L,\overline{\psi}_{\alpha}} \\ &+ (1 - \lambda) \left[F_{L}(\overline{\psi}_{\alpha})u(w_{0} - \pi_{L}D) - \int_{0}^{\overline{\psi}_{\alpha}} f_{L}(\psi)\psi d\psi - F_{L}(\overline{\psi}_{\alpha})V_{L}(\alpha_{L}) \right] \\ &= (1 - \lambda) \left[F_{L}(\overline{\psi}_{\alpha})\left(u(w_{0} - \pi_{L}D) - V_{L}(\alpha_{L})\right) - \int_{0}^{\overline{\psi}_{\alpha}} f_{L}(\psi)\psi d\psi \right] \\ &> 0 \end{split}$$

This holds by Definition 2.

(iv.ii):

$$\begin{split} V(\beta'_{L},\beta'_{H},\gamma_{L}) &- V(\beta_{L},\beta_{H}) < 0 \\ \Leftrightarrow &- (1-\lambda) \left[F_{L}(\overline{\psi}_{\beta}) \cdot (u(w_{0}-\pi_{L}D)-V_{L}(\beta_{L})) - \int_{0}^{\overline{\psi}_{\beta}} f_{L}(\psi)\psi d\psi \right] \\ &< \lambda \Delta_{\gamma} V_{H} + (1-\lambda)(1-F_{L}(\overline{\psi}_{\beta}))\Delta_{\gamma} V_{L,\overline{\psi}_{\beta}} \\ \Leftrightarrow &- (1-\lambda) \left[F_{L}(\overline{\psi}_{\beta}) \cdot (u(w_{0}-\pi_{L}D)-V_{L}(\beta_{L})) - \int_{0}^{\overline{\psi}_{\beta}} f_{L}(\psi)\psi d\psi \right] \\ &< \lambda \left[V_{H}(\beta'_{H}) - V_{H}(\beta_{H}) \right] + (1-\lambda)(1-F_{L}(\overline{\psi}_{\beta})) \left[V_{L}(\beta'_{L}) - V_{L}(\beta_{L}) \right] \\ \Leftrightarrow (1-\lambda) \left[F_{L}(\overline{\psi}_{\beta}) \cdot (u(w_{0}-\pi_{L}D)-V_{L}(\beta_{L})) - \int_{0}^{\overline{\psi}_{\beta}} f_{L}(\psi)\psi d\psi \right] \\ &> \lambda \left[V_{H}(\beta_{H}) - V_{H}(\beta'_{H}) \right] + (1-\lambda)(1-F_{L}(\overline{\psi}_{\beta})) \left[V_{L}(\beta_{L}) - V_{L}(\beta'_{L}) \right] \end{split}$$

$$\begin{aligned} V(\beta'_L, \beta'_H, \gamma_L) - V(\beta_L, \beta_H) &> 0 \\ \Leftrightarrow (1 - \lambda) \left[F_L(\overline{\psi}_\beta) \cdot (u(w_0 - \pi_L D) - V_L(\beta_L)) - \int_0^{\overline{\psi}_\beta} f_L(\psi) \psi d\psi \right] \\ &< \lambda \left[V_H(\beta_H) - V_H(\beta'_H) \right] + (1 - \lambda)(1 - F_L(\overline{\psi}_\beta)) \left[V_L(\beta_L) - V_L(\beta'_L) \right] \end{aligned}$$

The option to reveal private information in this context leads to a welfare gain if the aggregate increase in expected utility for low risks with low privacy concerns outweighs the aggregate expected utility loss for high risks and low risks with high privacy concerns. However, if the aggregate expected utility loss for high risks and low risks with high privacy concerns outweighs the aggregate increase in expected utility for low risks with low privacy concerns, the availability of a screening contract leads to a welfare loss. (iv.iii):

$$\begin{split} V(\alpha_{H}, \alpha_{L}, \gamma_{L}) - V(\beta_{H}, \beta_{L}) &< 0 \\ \Leftrightarrow - (1 - \lambda) \left[F_{L}(\overline{\psi}_{\beta}) \cdot (u(w_{0} - \pi_{L}D) - V_{L}(\beta_{L})) - \int_{0}^{\overline{\psi}_{\beta}} f_{L}(\psi)\psi d\psi \right] \\ &< \lambda \Delta_{\gamma} V_{H} + (1 - \lambda)(1 - F_{L}(\overline{\psi}_{\beta}))\Delta_{\gamma} V_{L,\overline{\psi}_{\beta}} \\ \Leftrightarrow - (1 - \lambda) \left[F_{L}(\overline{\psi}_{\beta}) \cdot (u(w_{0} - \pi_{L}D) - V_{L}(\beta_{L})) - \int_{0}^{\overline{\psi}_{\beta}} f_{L}(\psi)\psi d\psi \right] \\ &< \lambda \left[V_{H}(\alpha_{H}) - V_{H}(\beta_{H}) \right] + (1 - \lambda)(1 - F_{L}(\overline{\psi}_{\beta})) \left[V_{L}(\alpha_{L}) - V_{L}(\beta_{L}) \right] \\ \Leftrightarrow (1 - \lambda) \left[F_{L}(\overline{\psi}_{\beta}) \cdot (u(w_{0} - \pi_{L}D) - V_{L}(\beta_{L})) - \int_{0}^{\overline{\psi}_{\beta}} f_{L}(\psi)\psi d\psi \right] \\ &> \lambda \left[V_{H}(\beta_{H}) - V_{H}(\alpha_{H}) \right] + (1 - \lambda)(1 - F_{L}(\overline{\psi}_{\beta})) \left[V_{L}(\beta_{L}) - V_{L}(\alpha_{L}) \right] \end{split}$$

$$\begin{split} V(\alpha_H, \alpha_L, \gamma_L) - V(\beta_H, \beta_L) &> 0 \\ \Leftrightarrow (1 - \lambda) \left[F_L(\overline{\psi}_\beta) \cdot (u(w_0 - \pi_L D) - V_L(\beta_L)) - \int_0^{\overline{\psi}_\beta} f_L(\psi) \psi d\psi \right] \\ &< \lambda \left[V_H(\beta_H) - V_H(\alpha_H) \right] + (1 - \lambda)(1 - F_L(\overline{\psi}_\beta)) \left[V_L(\beta_L) - V_L(\alpha_L) \right] \end{split}$$

The option to reveal private information in this context leads to a welfare gain if the aggregate gate increase in expected utility for low risks with low privacy concerns outweighs the aggregate expected utility loss for high risks and low risks with high privacy concerns. However, if the aggregate expected utility loss for high risks and low risks with high privacy concerns outweighs the aggregate increase in expected utility for low risks with low privacy concerns, the availability of a screening contract leads to a welfare loss. \Box

A.2.2 Proof of Proposition 5:

To show:

(i)
$$\Delta_{\psi} V_{H} \coloneqq \begin{cases} (i.i) & V_{H}(\alpha_{H}) - V_{H}(\alpha_{H}) = 0, & \text{if } \lambda_{RS} < \lambda_{\tau}, \tau \in \{\alpha, \beta\} \\ (i.ii) & V_{H}(\beta'_{H}) - V_{H}(\alpha_{H}) > 0, & \text{if } \lambda_{\beta} < \lambda_{RS} \end{cases}$$

$$(ii) \qquad \Delta_{\psi} V_{L,\overline{\psi}_{\beta}} \coloneqq \begin{cases} (ii.i) & V_L(\alpha_L) - V_L(\gamma_L) < 0, \quad \text{if } \lambda_{RS} < \lambda_{\tau}, \tau \in \{\alpha,\beta\} \\ (ii.ii) & V_L(\beta'_L) - V_L(\gamma_L) < 0, \quad \text{if } \lambda_{\beta} < \lambda_{RS} \end{cases}$$

(*iii*)
$$\Delta_{\psi} V_{L,\underline{\psi}_{\beta}} \coloneqq V_{L,\underline{\psi}_{\beta}}(\gamma_L) - V_L(\gamma_L) < 0$$

$$(iv) \qquad \Delta_{\psi}V \coloneqq \begin{cases} (iv.i) & V(\alpha_H, \alpha_L, \gamma_L) - V(\alpha_H, \gamma_L) < 0, & \text{if } \lambda_{RS} < \lambda_{\tau}, \tau \in \{\alpha, \beta\} \\ (iv.ii) & V(\beta'_L, \beta'_H, \gamma_L) - V(\alpha_H, \gamma_L) <> 0, & \text{if } \lambda_{\beta} < \lambda_{RS} \end{cases}$$

(i):

(i.i): The proof is obvious and therefore omitted.

(i.ii):

$$V_H(\beta'_H) - V_H(\alpha_H) < 0$$

$$\Leftrightarrow u(w_0 - p^{\beta'_H}) - u(w_0 - \pi_H D) < 0$$

$$\Leftrightarrow u(w_0 - p^{\beta'_H}) < u(w_0 - \pi_H D)$$

Since the utility function is increasing in wealth, this holds due to $\pi_H D > p^{\beta'_H}$.

 $\underline{(ii):}\;(ii.i)$ and (ii.ii) hold by Definition 2.

$$V_{L,\underline{\psi}_{\beta}}(\gamma_{L}) - V_{L}(\gamma_{L}) < 0$$

$$\Leftrightarrow u(w_{0} - \pi_{i}D) - \underline{\psi}_{\beta} - u(w_{0} - \pi_{i}D) < 0$$

$$-\underline{\psi}_{\beta} < 0$$

This holds due to Definition 1.

(iv): We make use of the definition

$$\Delta_{\psi}V \coloneqq \lambda \Delta_{\psi}V_{H} + (1-\lambda)(1 - F_{L}(\overline{\psi}_{\tau}))\Delta_{\psi}V_{L,\overline{\psi}_{\tau}} + (1-\lambda)F_{L}(\overline{\psi}_{\tau})\Delta_{\psi}V_{L,\underline{\psi}_{\tau}}, \ \tau \in \{\alpha,\beta\}.$$

(iv.i):

$$V(\alpha_{H}, \alpha_{L}, \gamma_{L}) - V(\gamma_{L}, \alpha_{H}) < 0$$

$$\Leftrightarrow \lambda \Delta_{\psi} V_{H} + (1 - \lambda)(1 - F_{L}(\overline{\psi}_{\alpha}))\Delta_{\psi} V_{L,\overline{\psi}_{\alpha}} + (1 - \lambda)F_{L}(\overline{\psi}_{\alpha})\Delta_{\psi} V_{L,\underline{\psi}_{\alpha}} < 0$$

$$\Leftrightarrow \lambda \Delta_{\psi} V_{H} + (1 - \lambda)(1 - F_{L}(\overline{\psi}_{\alpha}))\Delta_{\psi} V_{L,\overline{\psi}_{\alpha}} + (1 - \lambda) \cdot \left[-\int_{0}^{\overline{\psi}_{\alpha}} f_{L}(\psi)\psi d\psi\right] < 0$$

$$\Leftrightarrow (1 - \lambda)(1 - F_{L}(\overline{\psi}_{\alpha}))[V_{L}(\alpha_{L}) - V_{L}(\gamma_{L})] + (1 - \lambda) \cdot \left[-\int_{0}^{\overline{\psi}_{\alpha}} f_{L}(\psi)\psi d\psi\right] < 0$$

(iv.ii):

$$\begin{split} V(\beta'_{H},\beta'_{L},\gamma_{L}) - V(\alpha_{H},\gamma_{L}) &> 0 \\ \Leftrightarrow \lambda \Delta_{\gamma} V_{H} &> -(1-\lambda) \left[-\int_{0}^{\overline{\psi}_{\beta}} f_{L}(\psi)\psi d\psi \right] - (1-\lambda)(1-F_{L}(\overline{\psi}_{\beta}))\Delta_{\gamma} V_{L,\overline{\psi}_{\beta}} \\ \Leftrightarrow \lambda \left[V_{H}(\beta'_{H}) - V_{H}(\alpha_{H}) \right] &> -(1-\lambda) \left[-\int_{0}^{\overline{\psi}_{\beta}} f_{L}(\psi)\psi d\psi \right] \\ -(1-\lambda)(1-F_{L}(\overline{\psi}_{\beta})) \left[V_{L}(\beta'_{L}) - V_{L}(\gamma_{L}) \right] \\ \Leftrightarrow \lambda \left[V_{H}(\alpha_{H}) - V_{H}(\beta'_{H}) \right] &< (1-\lambda) \left[-\int_{0}^{\overline{\psi}_{\beta}} f_{L}(\psi)\psi d\psi \right] \\ +(1-\lambda)(1-F_{L}(\overline{\psi}_{\beta})) \left[V_{L}(\gamma_{L}) - V_{L}(\beta'_{L}) \right] \end{split}$$

(iii):

$$\begin{aligned} V(\beta'_H, \beta'_L, \gamma_L) - V(\alpha_H, \gamma_L) &< 0 \\ \Leftrightarrow \lambda \left[V_H(\alpha_H) - V_H(\beta'_H) \right] > (1 - \lambda) \left[-\int_0^{\overline{\psi}_\beta} f_L(\psi) \psi d\psi \right] \\ &+ (1 - \lambda)(1 - F_L(\overline{\psi}_\beta)) \left[V_L(\gamma_L) - V_L(\beta'_L) \right] \end{aligned}$$

The existence of privacy concerns in this context leads to a welfare gain if the aggregate increase in high risks' expected utility outweighs the aggregate expected utility loss for low risks. However, if the aggregate expected utility loss for low risks outweighs the aggregate increase in high risks' expected utility, privacy concerns lead to a welfare loss. \Box

A.3 Calculations for the Illustration of Changes in Consumers' Expected Utility

For the calculation of low-risk individuals' expected utility from a non-cross-subsidizing separating contract L, we need to derive the optimal coverage for a contract that does not attract high risks. The optimal coverage can be determined by the following maximization problem:

$$\max_{q_L} (1 - \pi_L) \cdot u(w_0 - p_L) + \pi_L \cdot u(w_0 - p_L + q_L - D)$$

s.t. $(1 - \pi_H) \cdot u(w_0 - p_L) \le u(w_0 - p_H) + \pi_H \cdot u(w_0 - p_L + qL - D)$
 $p_H = \pi_H D$
 $p_L = \pi_L q_L$
 $q_L \le D \le w_0$
 $0 < \pi_L < \pi_H < 1$
 $0 < \lambda < 1$

We use an alternative approach oriented at the graphical illustration: From Rothschild and Stiglitz (1976), we know that the fair odd lines are of the following form

$$Ei = -\frac{1 - \pi_i}{\pi_i}w_1 + n$$

with $i \in \{H, L\}$.

The position of the fair odd lines are derived as follows

$$w_0 - D = -\frac{1 - \pi_i}{\pi_i} w_0 + n$$
$$n = \frac{1}{\pi_i} w_0 - D$$

Therefore, the low risks' fair odd line is given by:

$$EL = -\frac{1 - \pi_L}{\pi_L} w_1 + \frac{1}{\pi_L} w_0 - D$$

Analogously, the high risks' fair odd line is given by:

$$EH = -\frac{1 - \pi_H}{\pi_H} w_1 + \frac{1}{\pi_H} w_0 - D$$

In order to derive the high risks' indifferent curve, we first solve their expected utility by the wealth state in case of an accident w_2 .

$$V_H = (1 - \pi_H)u(w_1) + \pi_H u(w_2)$$
$$u(w_2) = \frac{V_H - (1 - \pi_H)u(w_1)}{\pi_H}$$
$$w_2(w_1) = u^{-1} \left(\frac{V_H - (1 - \pi_H)u(w_1)}{\pi_H}\right)$$

The level of high risks' utility at full insurance for a fair premium is given at:

$$V_H(D) = u(w_0 - \pi_H D)$$

In order to derive the indifference curve for high risks at the expected level of the full insurance contract, we plug this expected utility level into $w_2(w_1)$:

$$w_2(w_1, V_H(D)) = u^{-1} \left(\frac{u(w_0 - \pi_H D) - (1 - \pi_H)u(w_1)}{\pi_H} \right)$$

Since the optimal contract for low risks has to make high risks indifferent to their fair contract with full insurance while still letting the insurer break even, the optimal q_L is found at the intersection of the high risks' indifference curve $w_2(w_1, V_H(D))$ and the low risks' fair odd line *EL*. Therefore, the optimal contract for low risks is implicitly defined by the following condition:

$$w_2(w_1, V_H(D)) = EL$$
$$u^{-1}\left(\frac{u(w_0 - \pi_H D) - (1 - \pi_H)u(w_1)}{\pi_H}\right) = -\frac{1 - \pi_L}{\pi_L}w_1 + \frac{1}{\pi_L}w_0 - D$$

For $u(\cdot) = ln(\cdot)$, we get

$$w_2(w_1, V_H(D)) = exp\left(\frac{ln(w_0 - \pi_H D) - (1 - \pi_H)ln(w_1)}{\pi_H}\right)$$

and therefore the optimal level of wealth in the no accident state for the low risk contract in a non-cross-subsidizing separating equilibrium is implicitly given by

$$w_2(w_1, V_H(D)) = EL$$
$$exp\left(\frac{\ln(w_0 - \pi_H D) - (1 - \pi_H)\ln(w_1))}{\pi_H}\right) = -\frac{1 - \pi_L}{\pi_L}w_1 + \frac{1}{\pi_L}w_0 - D$$

With $\pi_H = 0.7$, $\pi_L = 0.4$, $w_0 = 10$, and D = 9, we get $w_1 \approx 8.9798$ and therefore $q_L = 2.5505$.